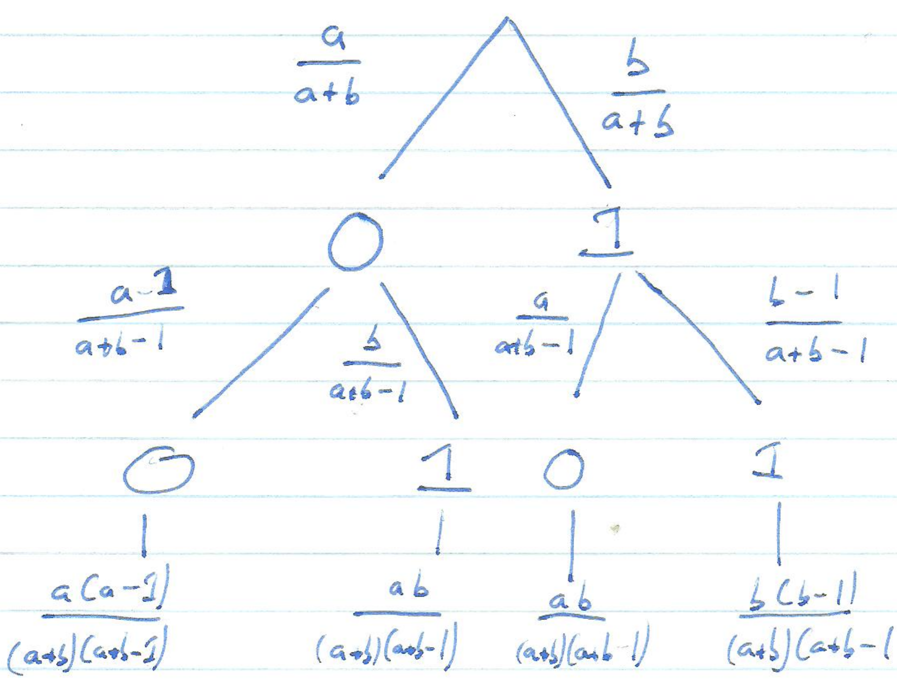
1.

So firstly here's a Bayesian Tree

~~Ea~~ ~~Note~~ Nodes marked "0" imply the selection of an even numbered ball and those marked "1", the selection of an odd numbered ball



2.

As Selections 0,1 and 1,0 result in odd sums and 0,0 and 1 1 result in even sums

If there is even chance

$$\Rightarrow \quad \frac{a(a-1)+b(b-1)}{(a+b)(a+b-1)} = \frac{2ab}{(a+b)(a+b-1)}$$

$$a^2 - a + b^2 - b = 2ab$$

$$\boxed{a^2 - (2b+1)a + b^2 - b = 0}$$

3.

$$a = \frac{2b+1 \pm \sqrt{8b+1}}{2}$$

for integer a $8b+1 = m^2$ for some int m.

4.

consider odd no. $2n+1$

then $(2n+1)^2 = 4n^2+4n+1$

$= 4(n^2+n)+1$

as $n^2+n = n(n+1) \overset{\text{and}}{\cancel{\Rightarrow}}$ $2 \mid n$ <u>or</u> $2 \mid (n+1)$

$\Rightarrow 2 \mid (n^2+n)$

and so $8 \mid 4(n^2+n)$

$4(n^2+n) = 8b$ for some positive int. b

and so using parameter

$(2m+1)^2 = 8b+1$

we can write the following equations for a and b

5, $b = \frac{\cancel{4}(m^2+m)}{\cancel{8}\ 2}$ (provides integers for all m)

$$a = \frac{m^2+m+1 \pm 2m+1}{\cancel{8}\ 2}$$

$a = \frac{m^2+3m+2}{\cancel{8}\ 2}$ $\quad$ $a = \frac{m^2-m}{\cancel{8}\ 2}$

$a = \frac{(m+2)(m+1)}{2}$ $\quad$ $a = \frac{m(m-1)}{2}$

6.

m	a	b	$a + b = m^2$
2	1	3	4
3	3	6	9
4	6	10	16
5	10	15	25
6	15	21	36
7	21	28	49
8	28	36	64
9	36	45	81
10	45	55	100
⋮	⋮	⋮	⋮