Step 1. Taking advantage of the fact that $(a+b)(a-b)=a^2-b^2$, We can rationalize the denominator.

As each fraction is in the form $\frac{1}{\sqrt{a}+\sqrt{b}}$ We can multiply each fraction by $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$ to give

$$\frac{1}{\sqrt{1}+\sqrt{2}} \times \frac{\sqrt{1}-\sqrt{2}}{\sqrt{1}-\sqrt{2}} \quad + \quad \frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} \; \ldots\ldots\ldots\ldots \; + \; \frac{1}{\sqrt{99}+\sqrt{100}} \times \frac{\sqrt{99}-\sqrt{100}}{\sqrt{99}-\sqrt{100}} \quad =$$

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$$\frac{\sqrt{1}-\sqrt{2}}{1-2} \quad + \quad \frac{\sqrt{2}-\sqrt{3}}{2-3} \; \ldots\ldots\ldots\ldots \; + \quad \frac{\sqrt{99}-\sqrt{100}}{99-100} \quad =$$

$$\frac{\sqrt{1}-\sqrt{2}}{-1} \quad + \quad \frac{\sqrt{2}-\sqrt{3}}{-1} \; \ldots\ldots\ldots\ldots \; + \quad \frac{\sqrt{99}-\sqrt{100}}{-1} \quad =$$

This can be written in sigma notation as

$$\sum_{n=1}^{99} -(\sqrt{n} - \sqrt{n+1}) \qquad \text{or} \qquad \sum_{n=1}^{99} -\sqrt{n} + \sqrt{n+1} \quad =$$

$$-\sqrt{1}+\sqrt{2} \quad -\sqrt{2}+\sqrt{3} \quad -\sqrt{3}+4 \quad -\sqrt{4}+\sqrt{5}\ldots\ldots\ldots\ldots \quad -\sqrt{98}+\sqrt{99} \quad -\sqrt{99}+\sqrt{100}$$

We see that we get a lot of cancellations and are left with

$$-\sqrt{1} + \sqrt{100} =$$
$$-1 \; + \; 10 \; = \; 9$$