$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{98}+\sqrt{99}}+\frac{1}{\sqrt{99}+\sqrt{100}}$$

$$=\frac{1(\sqrt{2}-\sqrt{1})}{(\sqrt{2}+\sqrt{1})(\sqrt{2}-\sqrt{1})}+\frac{1(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}+\frac{1(\sqrt{4}-\sqrt{3})}{(\sqrt{4}+\sqrt{3})(\sqrt{4}-\sqrt{3})}+\cdots+\frac{1(\sqrt{99}-\sqrt{98})}{(\sqrt{99}+\sqrt{98})(\sqrt{99}-\sqrt{98})}+\frac{1(\sqrt{100}-\sqrt{99})}{(\sqrt{100}+\sqrt{99})(\sqrt{100}-\sqrt{99})}$$

$$=\frac{\sqrt{2}-\sqrt{1}}{2-1}+\frac{\sqrt{3}-\sqrt{2}}{3-2}+\frac{\sqrt{4}-\sqrt{3}}{4-3}+\cdots+\frac{\sqrt{99}-\sqrt{98}}{99-98}+\frac{\sqrt{100}-\sqrt{99}}{100-99}$$

$$=(\sqrt{2}-\sqrt{1})+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+\cdots+(\sqrt{99}-\sqrt{98})+(\sqrt{100}-\sqrt{99})$$

$$=\sqrt{100}-1$$

$$=10-1$$

$$=9$$

1.The denominators contain square roots, so they need to be rationalized. Because (a + b) (a - b) = $a^{2} - b^{2}$, for the first fraction, we can multiply both the numerator and denominator by ($\sqrt{2}$ - 1), the second fraction by ($\sqrt{3}$ - $\sqrt{2}$) and so on.

2. The denominator of the first fraction will become 2 – 1, the second one 3 – 2 and so on.

3. The denominators all becomes 1!

4. At last we are left with $\sqrt{100}$ - 1, which is 9.