Niharika Paul

Age 10.

Marbles in a 3 box.

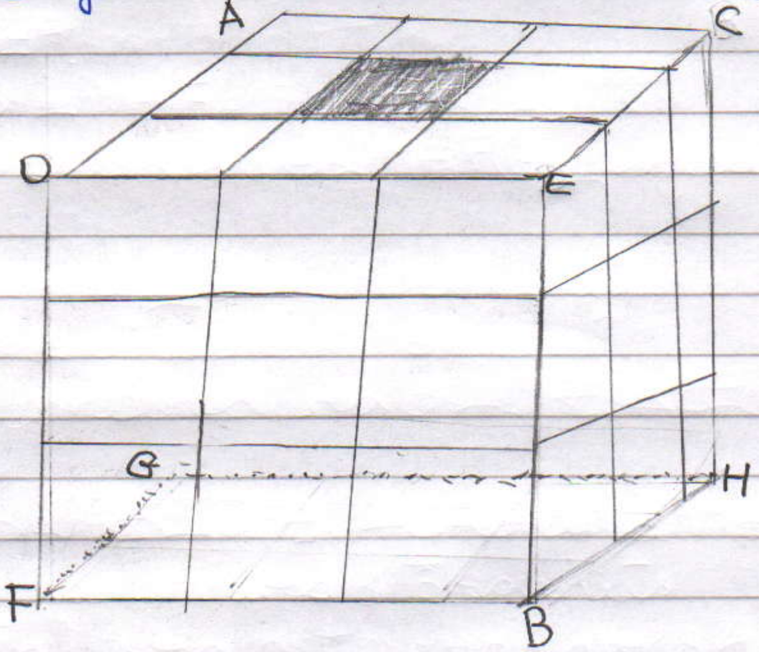
~~On a~~

Each of the edges is one of the winning lines. ⇒ We have found 12 winning lines already.

On each of the faces there are 8 winning lines but we are double-counting the four edges of each face from the previous step.

∴ We say that on each face, there are 4 lines. ~~So,~~

As there are 6 faces there will be 24 winning lines from the faces.



There will be one winning line from the shaded square to the its opposite square on the opposite face and as there are three pairs of opposite faces, there are 3 of these kinds of winning lines.

Between A and B there is one winning line. As there are ~~12~~ 8 vertices, there are ~~12~~ 8 $\div 2 =$ ~~6~~ 4 of these lines.

Between the face ACDE and GHFB, there is another face which has two winning lines - the diagonals to contribute. As there are 3 of these faces there are 6 of these altogether.

∴ The number of lines is $12+24+3+\cancel{6}\,4+6=\cancel{51}\,49$

Caroline's method - This is basically my method.
James' method - I like this method because it's very short.
Alison's method - This is the most generic.
Grae's method - This is too complicated for me!

This is the number of lines for an $n \times n \times n$ cube.
There will firstly be 12 edges.
Then there are $12 + 12(n-2) = 12(n-1)$ on the faces.
Then between two opposite faces there are $(n-2)^2 \Rightarrow$ altogether there are $3(n-2)^2$ of these lines.
Then there are 4 of the diagonals that go through the centre of the cube.
There are $(n-2)$ faces between two opposite faces on the surface. Each faces gives two lines and there $3(n-2)$ of these faces $\Rightarrow$ there are $6(n-2)$ of these lines.
$\therefore$ Altogether there are $12 + 12(n-1) + 3(n-2)^2 + 4 + 6(n-2) = 3n^2 + 6n + 4$.