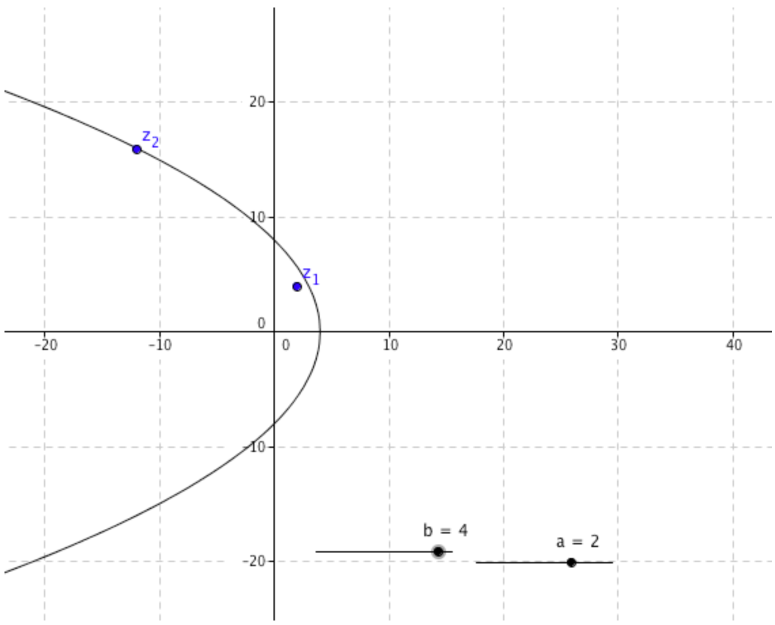
If we say that **z + yi** (Z_2) is the square of **a +bi** (Z_1):

$$z+yi = (a+bi)^2$$
$$z+yi = a^2+2abi+(bi)^2$$
$$z+yi = a^2+2abi+b*-1$$
$$z+yi = a^2+2abi-b^2$$

We can draw from this that $z = a^2 - b^2$ and $y = 2ab$
If we substitute $b = y/(2a)$ into the first equation we get:

$$z = a^2 – y^2/(4a^2)$$
$$4za^2 = 4a^4 – y^2$$
$$y^2 = 4a^4 - 4za^2$$
$$y = (4a^4(a^2-z))^{0.5}$$
$$y = 2a(a^2 – z)^{0.5}$$

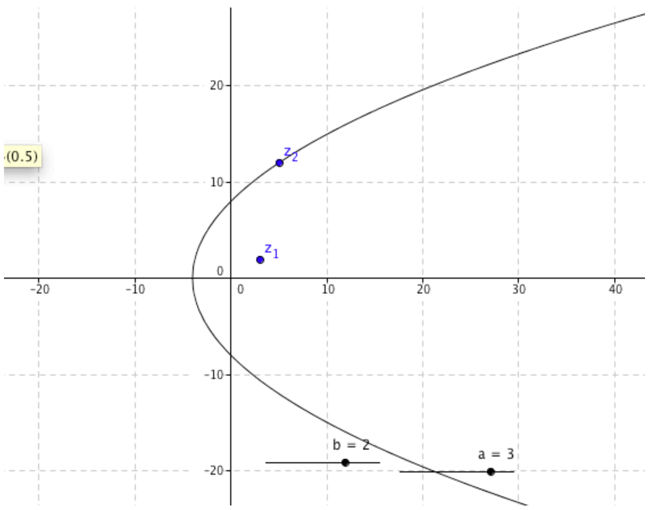
We can then plug this into an Argand diagram for any value of a:

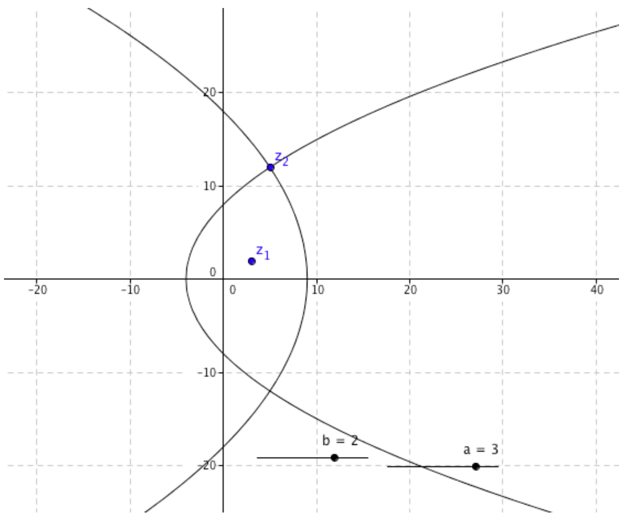


What we in effect get is all the different values of Z_2 for changing values of b (we also have to reflect the equation for when b is negative).

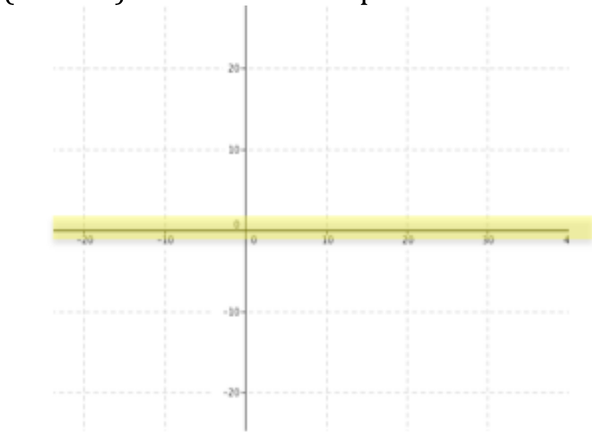
We can also do the same for changing values of b, which rearranges to:

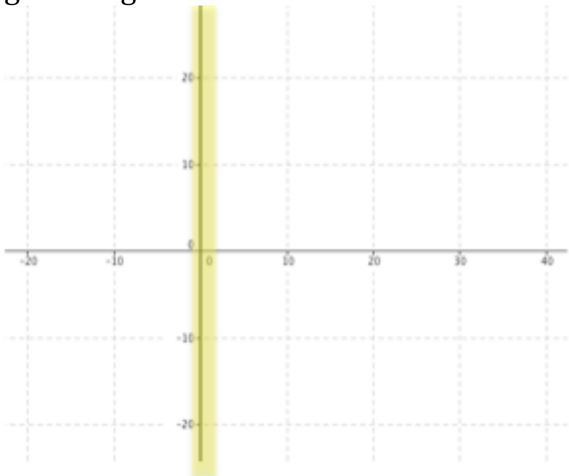
$$y = 2b(b^2 + z)^{0.5}$$





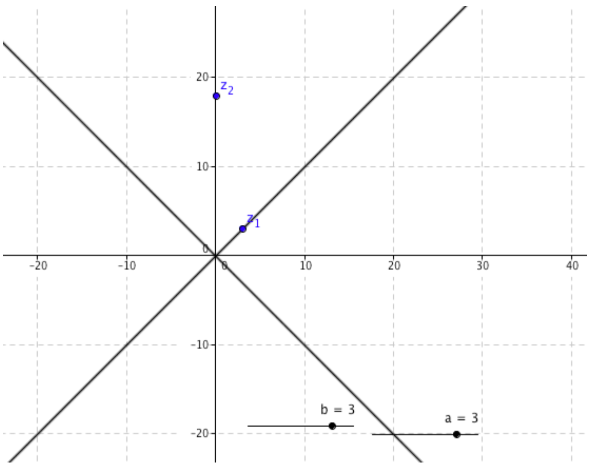
Z_2 will only be real when **a** or **b** is 0. This makes sense as, as shown earlier **z = a^2 - b^2 and y = 2ab** so for y to be zero (since it is the coefficient of i) either **a** or **b** (or both) must be zero. Represented or an Argand diagram:

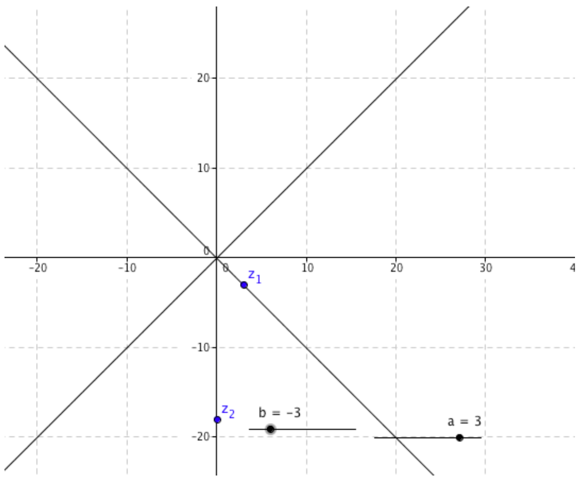




When **a** is zero, **a^2+2abi-b^2** is reduced to **-b^2** and when **b** is zero it is reduced to **a^2**. This means if **b** is zero the squared number will be positive and if **a** is zero it will be negative. In other words any positive number has a complex root with **b** equaling zero and **a** equaling the square root of the number and for a negative number vice versa.

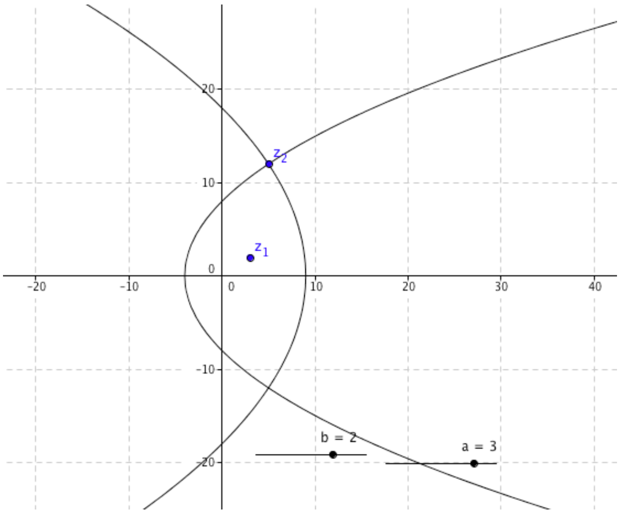
For Z_2 to be imaginary **a** and **b** have to be equal or the negative version of the other. This is because the real part of **a^2+2abi-b^2** is **a^2 - b^2** so for this to equal zero, **a^2** must equal **b^2.** On an Argand diagram:





If **a** and **b** are equal, **2ab** will be positive and so the imaginary coefficient will be positive. However, if one is the negative version of the other **2ab** will be negative and so the coefficient will be too. This means any positive imaginary number, **+yi**, can be represented as $(y^{0.5} + y^{0.5}i)^2$ or $(-y^{0.5} -y^{0.5}i)^2$ and any negative imaginary number **–yi** can be represented by $(-y^{0.5} + y^{0.5}i)^2$ or $(y^{0.5} - y^{0.5}i)^2$.

To return to the original graph drawn, the shape can be easily explained. If **b** is constant in **a^2+2abi-b^2** then as **2ab** increases (the y coordinate), **a^2** (the z coordinate) will increase at a polynomial degree 2 rate which is essentially the same as a $y=x^{0.5}$ graph hence the shape of the graph. When **a** is constant, the thing changing the z coordinate is **$-b^2$** hence the $y = -x^{0.5}$ shape.



To work out which quadrant a Z_2 will be in you first look at whether **a** and **b** are the same signs. If they are the same, **2ab** will always be positive so Z_2 will be in one of the top two quadrants. Since the coefficient of **c** is **a^2 - b^2,** Z_2 will will be on the right if **a** is further away from zero than **b** and Z_2 will be on the left if vice versa.

Can some Complex numbers square to give numbers that can then square to become real?

A number can square to have a or b equal 0 (as shown earlier)

Can some Complex numbers square to give numbers that can then square to become imaginary?

As discussed earlier for a complex number to square to become imaginary, **a** and and **b** must be equal → m+mi. This means in the original number 2ab = m and $a^2 - b^2 = m$. To check if this is possible I substituted one into the other to end up with:

$$m^2/(4b^2) - b^2 = m$$
$$4b^4 + 4mb^2 - m^2 = 0$$

I then treated this as a hidden quadratic and checked the discriminant:

$$(4m)^2 - 4x4x\text{-}m^2$$
$$= 16m^2 + 16m^2$$
$$= 32m^2$$

This shows that for any value of m there are 2 possible complex numbers that bring you to m +mi. A complex number can also square to become imaginary if **a** is the negative version of **b** → m-mi. Using the same method as before I ended up with this discriminant:

$$16m^2 - 16m^2$$
$$= 0$$

This means that for any complex number represented by m – mi there is 1 number that can square to bring you this value.

Footnote

If you draw a graph of $y = (a+b)^x$, you get some pretty cool shapes, which I guess have similar roots to mandelbrots: