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Double Trouble.

I see that:

$$\frac{1}{2}+\frac{1}{4}=\frac{3}{4}=1-\frac{1}{4}.$$

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=\frac{7}{8}=1-\frac{1}{8}.$$

I guess that $\frac{1}{2}+\frac{1}{4}\ldots\frac{1}{2^n}=1-\frac{1}{2^n}$

This ties in with Charlie's construct as when he draws 2 figures there is 1/4 of the square remaining and $1-\frac{1}{4}$ filled. When he draws 3 figures there is 1/8 of the square empty and $1-\frac{1}{8}$ drawn.

Now, what happens when we change the 2 to 3?

$\frac{1}{3}+\frac{1}{9}=\frac{4}{9}$ but this is not $1-\frac{1}{9}$.

$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}=\frac{13}{27}$ and neither is this $1-\frac{1}{9}$

However, $\frac{4}{9}=\left(1-\frac{1}{9}\right)\div 2$ and $\frac{13}{27}=\left(1-\frac{1}{27}\right)\div 2$.

I conject that $$\sum_{i=1}^{n}\frac{1}{3^i}=\frac{1-\frac{1}{3^i}}{\frac{1}{\frac{1}{3-1}}}=\frac{1-\frac{1}{3^i}}{2}$$

I generalise and guess that when $\{x_i\}_{i=1}^{n}$ and the ratio between 2 consecutive terms $=a$

Then, $$\sum_{i=1}^{n}x_i=\frac{1-a^n}{\frac{1}{a}-1}$$

Now I am going to proove this statement using induction.

I have already tried this for small values of a and n

I assume that this statement is true.

My aim is to prove that $\sum_{i=1}^{n+1} x_i = \frac{1-a^{n+1}}{\frac{1}{a}-1}$.

$$\sum_{i=1}^{n+1} x_i = \sum_{i=1}^{n} x_i + x_{n+1} = \frac{1-a^n}{\frac{1}{a}-1} + a^{n+1} = \frac{1-a^n + a^{n+1}\left(\frac{1}{a}-1\right)}{\frac{1}{a}-1}$$

$$= \frac{1-a^n+a^n-a^{n+1}}{\frac{1}{a}-1} = \frac{1-a^{n+1}}{\frac{1}{a}-1}$$

$$\therefore \sum_{i=1}^{n} x_i = \frac{1-a^n}{\frac{1}{a}-1}$$

QED

In Alison's sequence $a=2$.

$\therefore$ The sum of the first n terms of the sequence $=$

$$\frac{1-2^n}{\frac{1}{2}-1} = \frac{2^n-1}{\frac{1}{2}} = (2^n-1)2 = 2^{n+1}-2.$$

On top of this I had found a more 'cute' approach to finding the sum.

$$2+4+8\ldots 2^n = 2^{n+1}\left(\frac{1}{2^n}+\frac{1}{2^{n-1}}\ldots+\frac{1}{2}\right)$$

$$= 2^{n+1}\left(\frac{2^n-1}{2^n}\right)$$

$$= 2\,(2^n-1)$$

$$= 2^{n+1}-2$$