

Student 1:

By expanding $a^2 - x^2 = c^2 - (b - x)^2$ we get:

$$a^2 - x^2 = c^2 - b^2 + 2bx - x^2$$

Adding x^2 to both sides:

$$a^2 = c^2 - b^2 + 2bx$$

Rearrange for c :

$$c^2 = a^2 + b^2 + 2bx$$

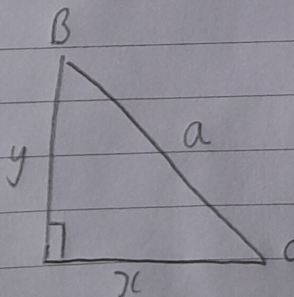
$$c = \sqrt{a^2 + b^2 + 2bx}$$

Substitute $x = a \cos C$:

$$c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

We have a formula for one side given we know the other two and the angle between them.

Our method only works when C is acute since our initial construction ~~relied on~~ involved a right angle triangle involving C and the angles in a right triangle must be less than 90 .



$$90 + B + C = 180$$

$$B + C = 90$$

$\therefore$

$$C < 90$$

Student 2:

c is the length between points B and A $\therefore$
 $\therefore (a, 0) = (x_1, y_1)$ and $(b \cos \theta, b \sin \theta) = (x_2, y_2)$

$$\Rightarrow c = \sqrt{(b \cos \theta - a)^2 + (b \sin \theta - 0)^2}$$

Expanding:

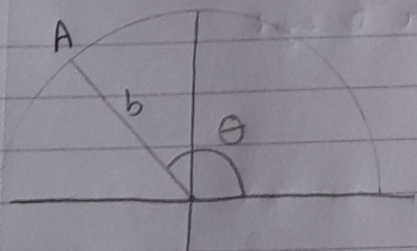
$$c = \sqrt{b^2 \cos^2 \theta - 2ab \cos \theta + a^2 + b^2 \sin^2 \theta}$$

We can use the identity $\sin^2 \theta + \cos^2 \theta \equiv 1$ to simplify:

$$c = \sqrt{a^2 + b^2 - 2ab \cos \theta}$$

This gives the same formula as student 1, only swapping θ for C , which are the same.

This method also works for all values of θ since point A can be thought of as being on the circumference of a circle with radius b .



A can always be defined as $(b \cos \theta, b \sin \theta)$

Student 3:

Following on from the student:

$$a^2 + b^2 = 2ab \cos C + b \cos A + a \cos B$$

Substituting $c^2 = b \cos A + a \cos B$

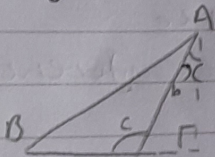
$$a^2 + b^2 = c^2 + 2ab \cos C$$

$$\Rightarrow c^2 = a^2 + b^2 - 2ab \cos C$$

$$\Rightarrow c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

This again gives the same formula.

The method is also only works for when C is obtuse since the construction to work out a and b relies on this



$180 - C$ is part of a right angle triangle

$$\therefore 180 = 90 + (180 - C) + x \text{ where}$$

x is the other angle in the triangle

$$\Rightarrow C = 90 + x$$

$\therefore C$ is obtuse