Graphs of Changing Areas

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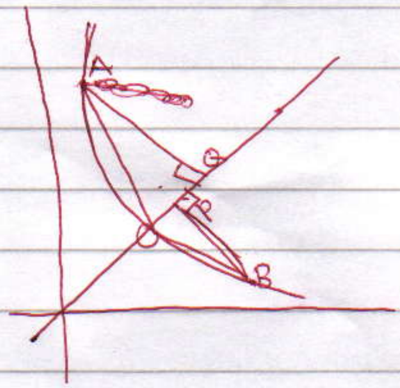
Age ~~8~~ 9

Every rectangle has the same area of 10 unit^2



I think that the symmetry line is $y=x$

$\therefore$ The point where $y=x$ and $y=\frac{10}{x}$ cut means that $\frac{10}{x}=x$

$\therefore$ When $\frac{10}{x}=x$, $x=\sqrt{10}$ and $y=\sqrt{10}$

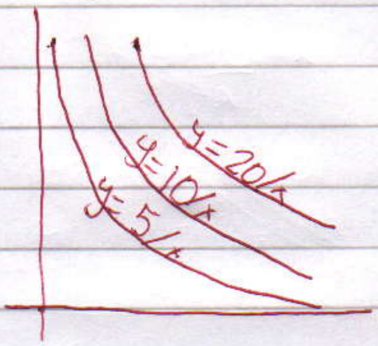
Now, I need to prove that $x=y$ is the sym. line. To to do this I ~~must~~ choose two points A and B. Where, $A\equiv(x_1,y_1)$ and $B\equiv(x_2,y_2)$. Here, $\cancel{\sqrt{10}-x_1=x_1-\sqrt{10}=\sqrt{10}-x_2 \text{ and}}$ $\sqrt{10}-x_1=x_2-\sqrt{10}$ and $y_1-\sqrt{10}=\sqrt{10}-y_2$

To prove that $x=y$ is the sym. line I must show that $\triangle AOQ \cong \triangle POB$.

Statement	Reason
QO common	Given
$\angle AQO = \angle OPB = 90$	By construct
AO = OB	I have chosen A, B so that this is true.

I will use the general case of $y = \frac{n}{x}$.

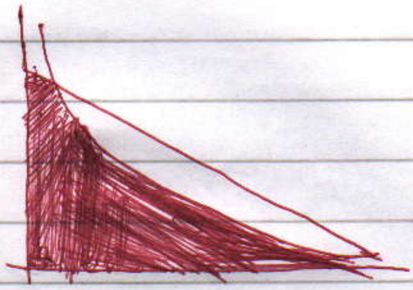
Where, n is $\in \mathbb{R}$, n is a constant.
To plot this function I will use the wise choice of $x=1$. At this point $y=n$.
Using this I will plot $y = \frac{20}{x}$, $y = \frac{10}{x}$ and $y = \frac{5}{x}$.



The lines can never intersect because

$\frac{n}{x} = \frac{m}{x}$

If $n=m$.



The straight line is $\frac{1}{2}P - x$. It will not cut $\frac{10}{x}$ if it lies in shaded region. I will confirm that $\frac{1}{2}P - x$ cuts $\frac{10}{x}$ with algebra.

Let $\frac{1}{2}P = k$.

$\therefore \quad k - x = \frac{10}{x}$

~~or $10 - kx^2 = 0$~~

or $x^2 - kx + 10 = 0$

or $\left(x - \frac{k}{2}\right)^2 + 10 - \frac{k^2}{4} = 0$

$$\text{or } \left(x-\frac{k}{2}\right)^2 - \left(\frac{k^2}{4}-10\right) = 0$$

$$\text{or } \left(x-\frac{k}{2}\right)^2 - \left(\sqrt{\frac{k^2}{4}-10}\right)^2 = 0$$

$$\text{or } \left(x-\frac{k}{2}+\sqrt{\frac{k^2}{4}-10}\right)\left(x-\frac{k}{2}-\sqrt{\frac{k^2}{4}-10}\right)$$

$$\therefore x = \frac{k}{2} - \sqrt{\frac{k^2}{4}-10} \text{ or } x = \frac{k}{2} + \sqrt{\frac{k^2}{4}-10}$$

If $\frac{k^2}{4} < 10$ then $x \notin \mathbb{R}$

i.e. if $k^2 < 40$ then $y = \frac{10}{x}$ and $y = \frac{P}{2} - x$ will not cut.

or $k < \sqrt{40}$

$\therefore \frac{P}{2} < 2\sqrt{40}$

or $P < 4\sqrt{10}$

$\therefore$ The shaded region is the region where $P < 4\sqrt{10}$.

$\therefore$ The smallest possible perimeter of rectangles with area of $10 = 4\sqrt{10}$.