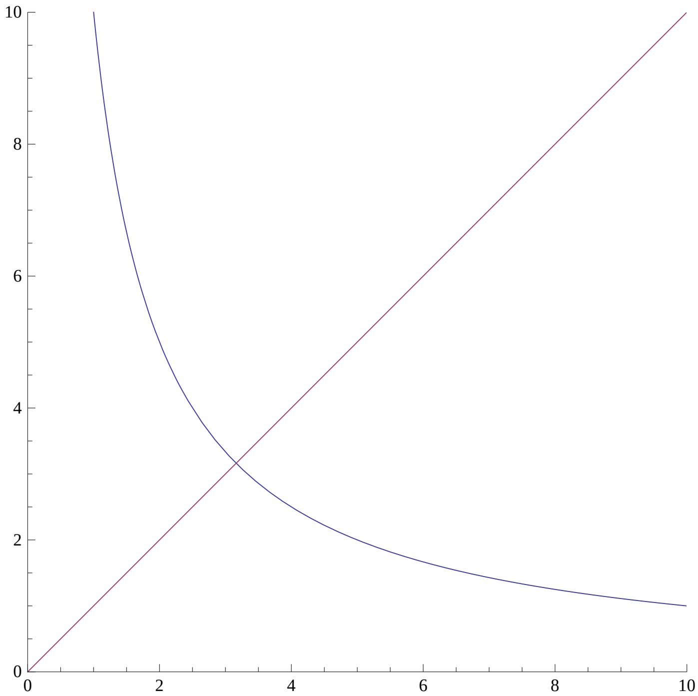
If $y == \frac{10}{x}$, then $x\,y == 10$ and so the rectangles all have area 10 square units.

The graph has reflective symmetry along the line $y == x$:

In[73]:=
```
Plot[{10 / x, x}, {x, 0, 10}, PlotRange → {{0, 10}, {0, 10}}, AspectRatio → 1]
```

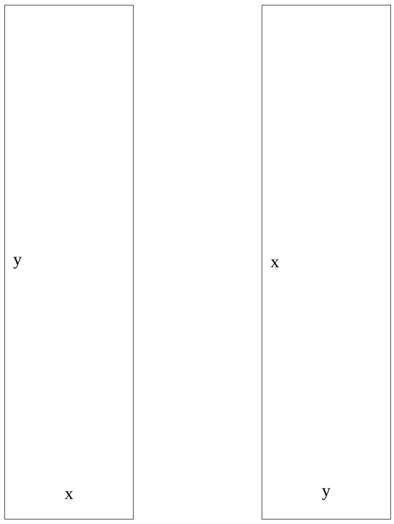
Out[73]=



since on any rectangle we can just relabel y and x to get another satisfactory rectangle:

In[136]:=
```
Graphics[{
  EdgeForm[Directive[Thin]], White, Rectangle[{0, 0}, {1, 4}],
  Black, Text["x", {0.5, 0.2}],
  Text["y", {0.1, 2}],
  EdgeForm[Directive[Thin]], White, Rectangle[{2, 0}, {3, 4}],
  Black, Text["y", {2.5, 0.2}],
  Text["x", {2.1, 2}]
 }]
```

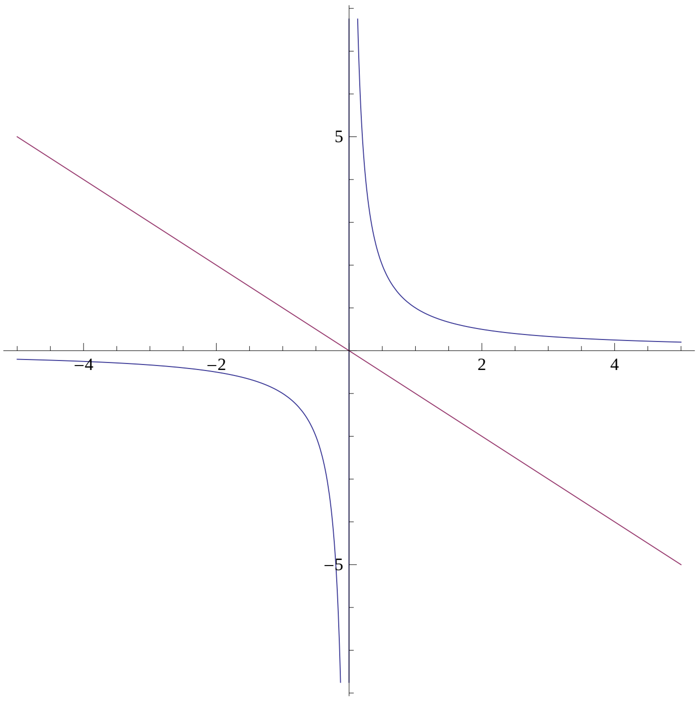
Out[136]=



As x increases, y tends to 0, since this represents a rectangle getting bigger and bigger in one direction and so smaller and smaller in the other (to maintain the same area). This is true of all equations of the form $y == a/x$.
The intersection with $y == x$ represents a square (a rectangle with both pairs of opposite sides equal in length) - so $x == a/x$ and $x^2 == a$, so this intersection is at $\left(\sqrt{a}\,,\ \sqrt{a}\,\right)$. There can be no intersection of two curves with different a, since this would imply two rectangles with the same side lengths but different area, which is impossible.

The second part, with rectangles of equal perimeter, we have $y == \frac{P}{2} - x$ which is a straight line, gradient -1, intersecting the y axis at $\frac{P}{2}$. Since the line $y == a/x$ to represent rectangles is defined only in the upper right quadrant of the graph, there must be values for P which mean the line passes below the curve (for example, $y == -x + \frac{-1}{2}$ will never enter the upper right quadrant). However, there is also a value which will never intersect $y == \frac{a}{x}$ for any non-zero real a: $P == 0 \Rightarrow y == -x$:

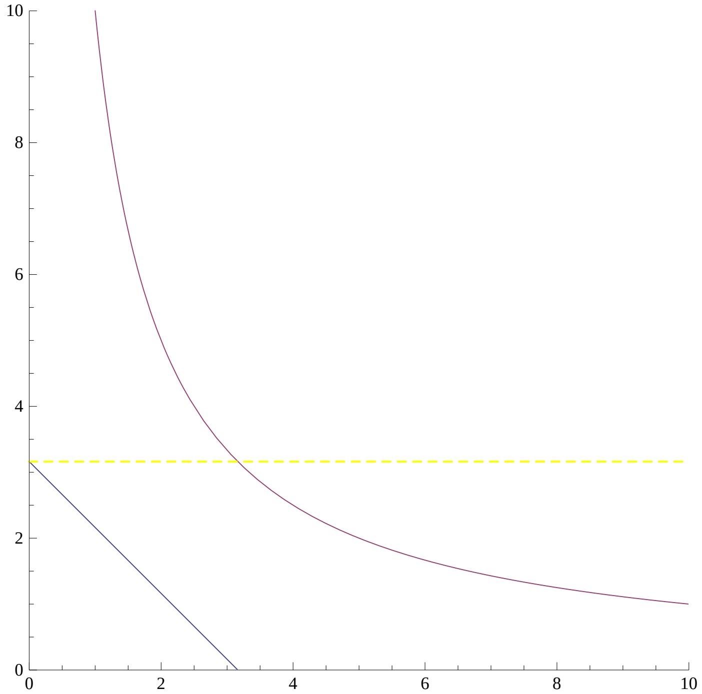
In[106]:=
```
Plot[{1 / x, -x}, {x, -5, 5}, AspectRatio → 1]
```

Out[106]=


Assuming P must be positive and therefore actually represents a valid rectangle, however, there will always exist an a such that the line $y == -x + \frac{P}{2}$ intersects $y == \frac{a}{x}$ (since we can move the apex of $\frac{a}{x}$ arbitrarily close to the origin by decreasing a): we have that for all P there exists a such that the lines intersect, and for all a there exists a valid P (intuitively, we can increase P so that the y-intercept increases until an intersection is created at $\left(\sqrt{a}, \sqrt{a}\right)$). However, to answer the question as stated, we do not have that there exists a such that for all P, the lines intersect; we can always take the line with gradient -1 passing through a point slightly below and to the left of the apex of the curve. Such a line is $y == -x + \sqrt{a}$ which has gradient -1 and intersects the y-axis level with the apex of the curve:

In[132]:=
```
Plot[{-x + √10, 10/x, √10}, {x, 0, 10}, AspectRatio → 1,
 PlotStyle → {Automatic, Automatic, Directive[Yellow, Thickness[0.003], Dashed]},
 PlotRange → {{0, 10}, {0, 10}}]
```

Out[132]=



To minimize the perimeter of the rectangle with area 10, we need to find the smallest P such that the graphs intersect with $a == 10$. We need the line $y == -x + \frac{P}{2}$ to intersect point $\left(\sqrt{10}, \sqrt{10}\right)$, since this is where the line is tangent to the curve (representing the square with side lengths $\sqrt{10}$). This intuitively gives perimeter = $4\sqrt{10}$, but we can also work this out by using the equation of a line with gradient -1 and point $\left(\sqrt{10}, \sqrt{10}\right)$ to give $y == 2\sqrt{10} - x$ and so $\frac{P}{2} == 2\sqrt{10}$ and $P == 4\sqrt{10}$. In general, using the same argument, a square contains the most area for a given perimeter.