Function pyramids

The first aim is to find $f(a,b)$

$$\begin{array}{ccc} & f(a,b) & \\ a & & b \end{array}$$

- To do this, we ~~must~~ should enter numbers to try to find a pattern
  - specifically integers.
- After entering some numbers, you begin to notice only powers of 2 worked e.g. gave integers

① $f(1,1)=0$ $\quad f(2,2)=2$ $\quad f(10,10)=6.64\ldots$

② $f(1,2)=1$ $\quad f(2,7)=3.81$

$f(1,3)=1.58$ $\quad$ etc.

③ $f(1,4)=2$

To see the pattern, it is a good idea to rewrite the numbers as powers of 2:

① $f(2^0, 2^0)=0$

② $f(2^0, 2^1)=1$

③ $f(2^0, 2^2)=2$

The function $f(a,b)$ appears to add the powers of a and b in the form 2^n.

→ This is further backed up by using random numbers e.g.

$f(32{,}768, 32) = f(2^{15}, 2^5) = 20$

The function therefore needs to find 'n' in $a=2^n$ and 'm' in $b=2^m$.
To do that:

(logathrims)

$n=\log_2 a \quad m=\log_2 b \rightarrow m+n=f(a,b)$

$m+n=\log_2 a+\log_2 b=\log_2(ab)$ $\quad$ a and b: both positive or both negative

simplify using laws of logathrims.

$\rightarrow f(a,b)=\log_2(a,b)$

$$\begin{array}{ccccc} & f(a,b) & x & f(b,c) & \\ a & & b & & c \end{array} \Rightarrow \begin{array}{ccccc} & & f(d,e) & & \\ & d & & e & \\ a & & b & & c \end{array}$$

The function is the same for both levels.

Using this, we can then easily choose numbers for the middle layer (that we would like to appear).

We can say that $f(a,b)=\log_2 ab=d$

$\rightarrow 2^d=ab \rightarrow \dfrac{2^d}{b}=a$

We can choose the number for d (middle layer), and b ($b\in\mathbb{R}$; $\neq 0$) and find a, making any combination possible. (required)
Use it multiple times to get the top number

$\therefore$

$\log_2 ab<0 \qquad 0<ab<1$

To make the middle layer negative, $\log ab<0$;

$0<ab<2^0$; $\quad 0<ab<1$

To make ab $0<ab<1$ a and b must either:

- Both be decimals (Both positive or both negative) <1
- Or a b, where $b=\dfrac{1}{x}$ and $x>a \rightarrow \dfrac{a}{x} \quad \dfrac{1}{x}=b$; $x>a$

$b=\dfrac{1}{x} \quad x>a \quad x>0.$

You can find a function to find the number at the top of the pyramid:

$$f(d,e) = \log_2(de) = \log_2(\log_2 ab \times \log_2 bc)$$

For $f(d,e) = 1$, $de = 2$

Sub in a and b $\quad \log_2 ab = \dfrac{2}{\log_2 bc}$

Therefore, to find values that give 1, input 2 random no. (both positive or both negative, $\neq 0$) and solve e.g.

$a = 7 \quad b = 12$

$$\log_2(7 \times 12) = \frac{2}{\log_2 7c} \qquad \frac{2}{\log_2 84} = \log_2 7c = 0.3128755\ldots$$

$$\frac{2^{ANS}}{7} = c = 0.1035\ldots$$

This can easily be used for any ~~whole~~ no.

$$\rightarrow \log_2 ab = \frac{2^{f(d,e)}}{\log_2 bc} \quad \text{e.g. if } f(d,e) = 5.$$

For the top no. to be negative, $\log_2 de < 0$; $0 < de < 1$

$$0 < \log_2 ab \log_2 bc < 1$$

(You can use this method for the middle layer: use just $\log_2 ab$ instead.)

Sub in values for a and b

$$0 < \log_2 bc < \frac{1}{\log_2 ab}$$

$$0 < \frac{2^{\frac{1}{\log_2 ab}}}{b} \quad \frac{2^{\frac{1}{\log_2 ab}}}{b} > c \quad \text{(For positive)}$$

$\log_2 ab > 0$ and $\log_2 bc > 0$

or $\log_2 ab < 0$ and $\log_2 bc < 0$

$$0 > \frac{2^{\frac{1}{\log_2 ab}}}{b} \quad \text{(For negative)}$$

$$c > \frac{2^{\frac{1}{\log_2 ab}}}{b}$$

~~If only subbing in a~~

e.g. $a = 5 \quad b = 7$

$$\frac{2^{\frac{1}{\log_2 35}}}{7} > c \Rightarrow 0.16352\ldots$$

$$\log_2 7c > 0 \Rightarrow 2^{-\log_2 7} < c$$

$$0.1635 > c > \frac{1}{7}$$