Firstly, lets write a function $b(x)$ to count the number of bacteria at a point in time (x is in minutes).

Since the bacteria double after every fixed interval of time, the function should be of the form $2^{\lfloor x \rfloor}$ ($\lfloor x \rfloor$ is the floor function, which outputs the greatest integer less than or equal to x).

And since the doubling happens every 30minutes, you want the ~~valeue~~ of the value of the floor function part to increase by 1 after x has increased by 30. So the floor function part will be $\lfloor \frac{x}{30} \rfloor$.

$$\therefore b(x) = 2^{\lfloor \frac{x}{30} \rfloor}$$

Next, lets make a function called $X(x)$ to count the number of molecules of X, x minutes after the start. Since 1 bacteria produces 1 molecule of X every minute, the number of molecules of X will be the sum of the number bacteria 1minute ago to the number of bacteria 10min ago (we don't need to go further back than that as X molecules decay after 10minutes).

$$\therefore X(x) = b(x-1) + b(x-2) + \dots + b(x-10)$$

$$= 2^{\lfloor \frac{x-1}{30} \rfloor} + 2^{\lfloor \frac{x-2}{30} \rfloor} + \dots + 2^{\lfloor \frac{x-10}{30} \rfloor}$$

After 24 hours, $24 \times 60 = 1440$ minutes will have elapsed.

So $X(1440) = 2^{\lfloor \frac{1439}{30} \rfloor} + 2^{\lfloor \frac{1438}{30} \rfloor} + \dots + 2^{\lfloor \frac{1430}{30} \rfloor}$

$= 2^{47} + 2^{47} + \dots + 2^{47}$

$= 10 \times 2^{47}$

$= 1.4 \times 10^{15}$

1.4×10^{15} molecules of X spread out across Rudolph's nose is definitely enough for a concentration of 10^{11} molecules of X per ml, so Rudolph's nose will be glowing again in time!