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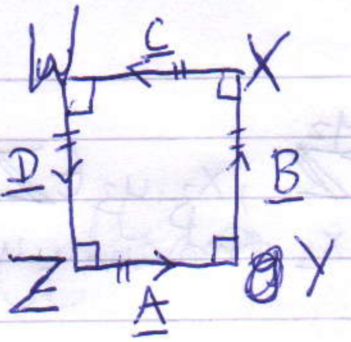
Q1.



~~A~~ Here we choose the sides of the square as vectors

Let: $\underline{A} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\quad x \in \mathbb{R}$, $y \in \mathbb{R}$

Then $\underline{B} = \begin{pmatrix} -y \\ x \end{pmatrix}$ or $\begin{pmatrix} y \\ -x \end{pmatrix}$

$\underline{C} = -\underline{A} = \begin{pmatrix} -x \\ -y \end{pmatrix}$

$\underline{D} = -\underline{B} = \begin{pmatrix} y \\ -x \end{pmatrix}$ or $\begin{pmatrix} -y \\ x \end{pmatrix}$

So we see ~~f~~ for a given $\underline{A}$ there are 2 possibilities for a square

Q2 If $\underline{A}$ joins the vertices Z and Y ~~which are g+~~

Let, $Z \equiv (x_1, y_1)$ and $Y \equiv (x_2, y_2)$ ~~x_1, y_1~~

$x_1, y_1, x_2, y_2 \in \mathbb{Z}$.

We want to prove that

If $X \equiv (x_3, y_3)$ and $W \equiv (x_4, y_4)$

Then $x_3, y_3, x_4, y_4 \in \mathbb{Z}$

~~Now,~~

Now, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

or $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$

or $x = x_2 - x_1$ and $y = y_2 - y_1$

Now, $x_2, x_1, y_2, y_1 \in \mathbb{Z} \Longrightarrow x, y \in \mathbb{Z}$

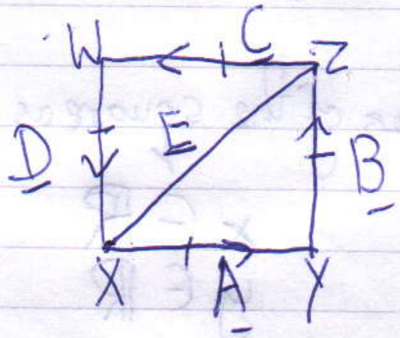
Now $\underline{B} = \begin{pmatrix} -y \\ x \end{pmatrix}$ or $\begin{pmatrix} y \\ -x \end{pmatrix}$

Case 1: $B = \begin{pmatrix} -y \\ x \end{pmatrix} = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} - \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

or, $-y = x_3 - x_2$ and $x = y_3 - y_2$

$x, y \in \mathbb{Z}^2$ and $x_2, y_2 \in \mathbb{Z}$, $x_3, y_3 \in \mathbb{Z}$

This also implies for $x = x_4$ ~~and~~ and $y = y_4$ and Case 2.



~~I need to find~~

Given: $|\underline{A}| = |\underline{B}|$

I need to find $\underline{A}$, $\underline{B}$ given $\underline{E}$

$\underline{E}$ at 45° to $\underline{A}$ and $\underline{B}$

~~$|\underline{E}|^2 =$~~

$\underline{E} = \underline{A} + \underline{B}$

Let $\underline{E} = \begin{pmatrix} x \\ y \end{pmatrix}$ $A = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ $B = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$$

$\underline{B} = \begin{pmatrix} -y_1 \\ x_1 \end{pmatrix}$ or $\begin{pmatrix} y_1 \\ -x_1 \end{pmatrix}$

Let $\underline{B} = \begin{pmatrix} -y_1 \\ x_1 \end{pmatrix}$

Then $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_1 \cancel{+} - y_1 \\ y_1 + x_1 \end{pmatrix}$

or $x = x_1 - y_1$ and $y = y_1 + x_1$

or $x = x_1 - y_1$

$-y = -y_1 - x_1$

or $x - y = \cancel{2y} = -2y_1$

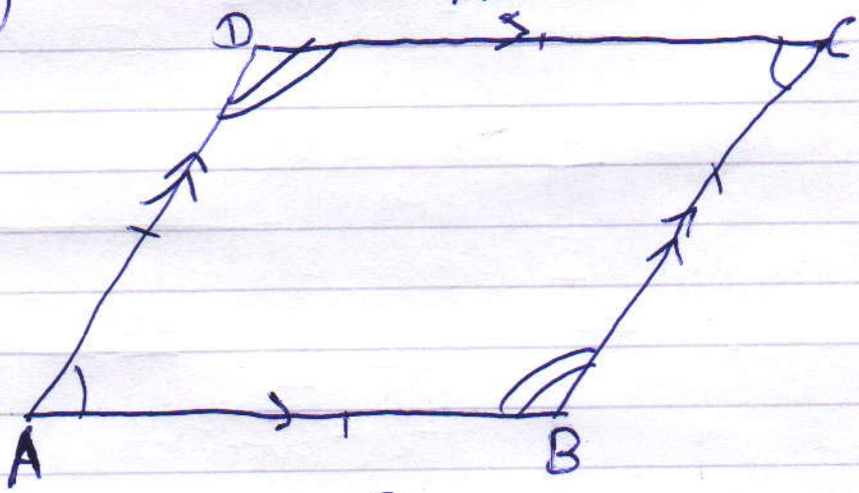
or $2y_1 = y - x$

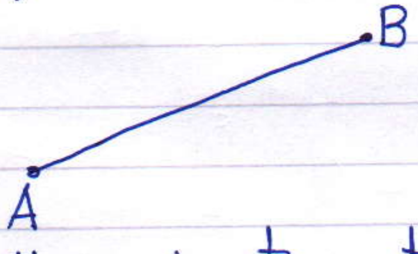
or $y_1 = \frac{y-x}{2}$

$x_1 = \frac{x+y}{2}$

(ii) We know x, y are integers. ~~because~~ However, x_1 and y_1 will not all ways be integer valued. It will be integer valued when $y-x$ and $x+y$ are even.

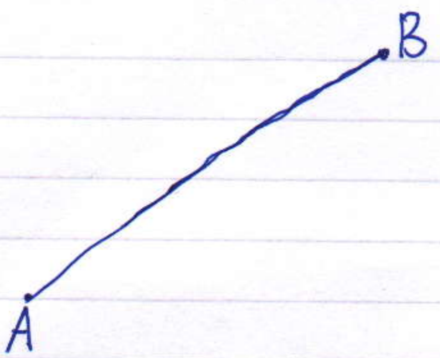
Charlie ~~is~~ is right ~~because~~ there is no fixed angle in a ~~rhom~~ rhombus' corner (angle ABC). So there are ∞ possibilities for the rhombus $\hat{ABCD}$.





We are going to construct a circle using A as the centre and AB as the radius. The number of points on the circle that lie on grid points are the number of rhombuses that have there vertices on grid points.

Yes I agree with Alison



We are going to draw another straight line called CD at 90° to AB. and then check ~~▒~~ if c and d are grid points. Then CD is the other diagonal of the rhombus ABCD.