

## Inner Equality

### Nrich Short Problem - Adithya Venkat (14)

The four numbers  $a, b, c, d$  lie between  $-5$  and  $5$ . In addition to this, the numbers are constrained further so that  $5 < a + b < 10$  and  $-10 < c + d < -5$ . Let's also suppose that  $0 < a, b < 5$  and  $-5 < c, d < 0$ .

$$1) ?? < a + b - c - d < ??$$

Firstly, I attempted to calculate the minimum and maximum values for  $-c - d$  using the inequality  $-10 < c + d < -5$ . Subtracting  $-d$  from  $-c$  can be a maximum of  $10$  if both  $c$  and  $d$  are equal to five:

$-(-5) - (-5) = 10$ . If  $c$  or  $d$  cannot equal zero at the same time, then the minimum value that  $-c - d$  has to be greater than is  $5$ , leaving the inequality  $5 < -c - d < 10$ . Due to the fact that  $5 < a + b < 10$ , we can add both minimum and maximum values of the two inequalities to give  $10 < a + b - c - d < 20$ .

$$2) ?? < a - c < ??$$

As previously established,  $0 < a < 5$  and  $0 < -c < 5$  therefore subtracting  $c$ , which lies between  $0$  and  $-5$ , from  $a$  can produce a minimum total of  $0$ , if both  $a$  and  $c$  are equal to  $0$ , and a maximum of  $10$ , occurring when both  $a$  and  $-c$  are equal to  $5$ . Thus, the inequality representing the subtraction of  $c$  from  $a$  is  $0 < a - c < 10$ .

$$3) ?? < a - c + d - b < ??$$

Since I have worked out the range of values for where  $a - c$  lies, I can substitute this inequality into the centre to give  $?? < (0 < a - c < 10) + d - b < ??$ . On the condition that we take the value  $b$  to equal a number between  $0$  and  $5$  whilst on the other hand, the value  $d$  to equal a number between  $-5$  and  $0$ , because in this instance I'm dealing with a negative  $b$  the minimum value for this particular subtraction would be  $-10$ , occurring when  $b = 5$  and  $d = -5$  as  $-5 - 5 = -10$ . This leaves just the maximum value to be calculated and this can be achieved through finding the two greatest numbers  $d$  and  $b$  can be equivalent to. Both negative  $b$  and  $d$  lie in between  $-5$  and  $0$  therefore the maximum value is  $0 + 0 = 0$ , giving  $-10 < d - b < 0$ . Combining the inequalities expressing the range of values where  $a - c$  and  $d - b$  lie gives  $-10 < a - c + d - b < 10$  because the sum of the minimum values is  $-10 + 0 = -10$  and the sum of the maximum values is  $0 + 10 = 10$ .

$$4) ?? < abcd < ??$$

Using the inequalities  $0 < a, b < 5$  and  $-5 < c, d < 0$ , I figured out that the maximum value for the product will be  $5 * 5 * -5 * -5 = 25^2 = 625$ . The minimum value for the product of  $abcd$  will be  $0$  as if either one of the numbers equals  $0$ , the product will be  $0$  due to the fact that any number multiplied by zero gives  $0$ .  $0 < abcd < 625$

$$5) ?? < \frac{|a| + |c|}{2} - \sqrt{|ac|} < ??$$

Substituting  $a = 0$  and  $c = -5$ , I was able to establish the minimum value for this inequality which is  $-5/2$  or  $-2.5$ . Substituting  $a = 5$  and  $c = 0$  gave me the maximum value which is  $5/2$  or  $2.5$  therefore leaving  $-2.5 < (a + c) \div 2 - \sqrt{ac} < 2.5$