Diminishing Returns

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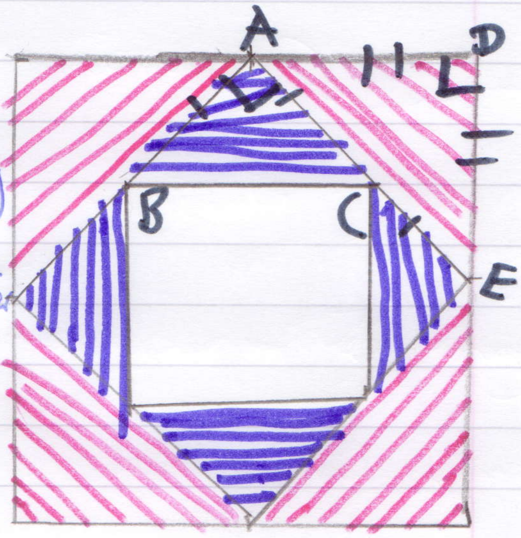
This is my first layer.



I shall compare only two triangles (ADE and ABC) as this is a matter of symmetry.

$AC \times 2 = AE$

$AE^2 = 2 \times (AD^2)$

$BC^2 = 2 \times (AC^2)$

Area of $\Delta ADE = \frac{AD^2}{2} = \frac{AE^2}{4}$

Area of $\Delta ABC = \frac{AC^2}{2}$

$\therefore$ Proportion of pink to blue ~~blue~~ $= \frac{\cancel{AE^2}}{\cancel{4}\,2} \times \frac{\cancel{2}}{\cancel{AC^2}} = 2$ ——— ①

$\therefore$ Proportion of blue in the first layer $= \frac{b}{p+b}$

$= \frac{b}{3b}$

$= \frac{1}{3}$

If we carry on this pattern ∞ times (as in Fig. 2) the proportion of blue will be $\frac{1}{3}$.

I' ~~fi~~figure 2 the proportion of blue is easier to calculate than in figure 1.

I expect that the proportion of blue in Figure 1 is slightly less than ~~in~~ $\frac{1}{3}$ because there is a pink square in the middle

The proportion of blue in Figure 1 is

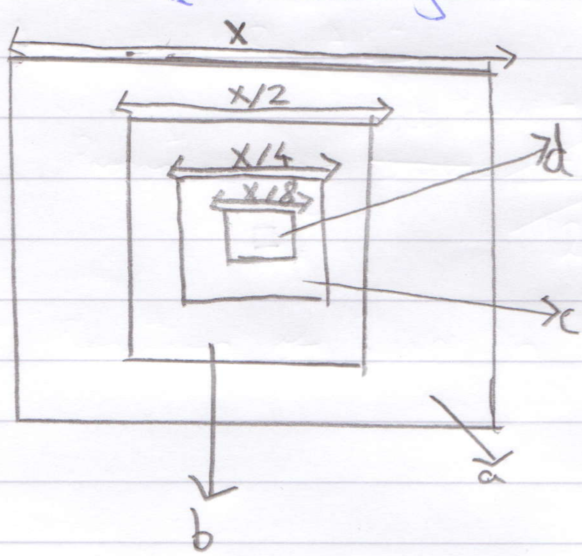
$$\frac{\frac{a+b+c}{3}}{a+b+c+d}$$

Where a = area of 1st Layer,

b = area of 2nd layer,

c = area of 3rd layer and

d = area of 4th layer.



$a = x^2 - \left(\frac{x}{2}\right)^2$ or

$x^2 - \frac{x^2}{4}$ or

$x^2\left(1 - \frac{1}{4}\right) = x^2\frac{3}{4}$

$b = \left(\frac{x}{2}\right)^2 - \left(\frac{x}{4}\right)^2$ or

$\frac{x^2}{4} - \frac{x^2}{16}$ or

$\frac{x^2}{4}\left(1 - \frac{1}{4}\right)$ or

$\frac{x^2}{4}\,\frac{3}{4}$ or

$\frac{3x^2}{16}$

$c = \left(\frac{x}{4}\right)^2 - \left(\frac{x}{8}\right)^2$ or

$\frac{x^2}{16} - \frac{x^2}{64}$ or

$\frac{x^2}{16}\left(1 - \frac{1}{4}\right)$ or

$\frac{x^2}{16}\,\frac{3}{4}$ or

$\frac{3x^2}{64}$

$d = \left(\frac{x}{8}\right)^2$ or

$\frac{x^2}{64}$

$\therefore$ Proportion of blue in figure 1:

$$\frac{\frac{3}{4}x^2 + \frac{3x^2}{16} + \frac{3x^2}{64} \cancel{+ \frac{x^2}{64}}}{\frac{3}{\frac{3}{4}x^2 + \frac{3x^2}{16} + \frac{3x^2}{64} + \frac{x^2}{64}}} =$$

$$\frac{1 + \frac{1}{4} + \frac{1}{16}}{3 + \frac{3}{4} + \frac{3}{16} + \frac{1}{16}} = \frac{21}{64} \text{ ans}$$