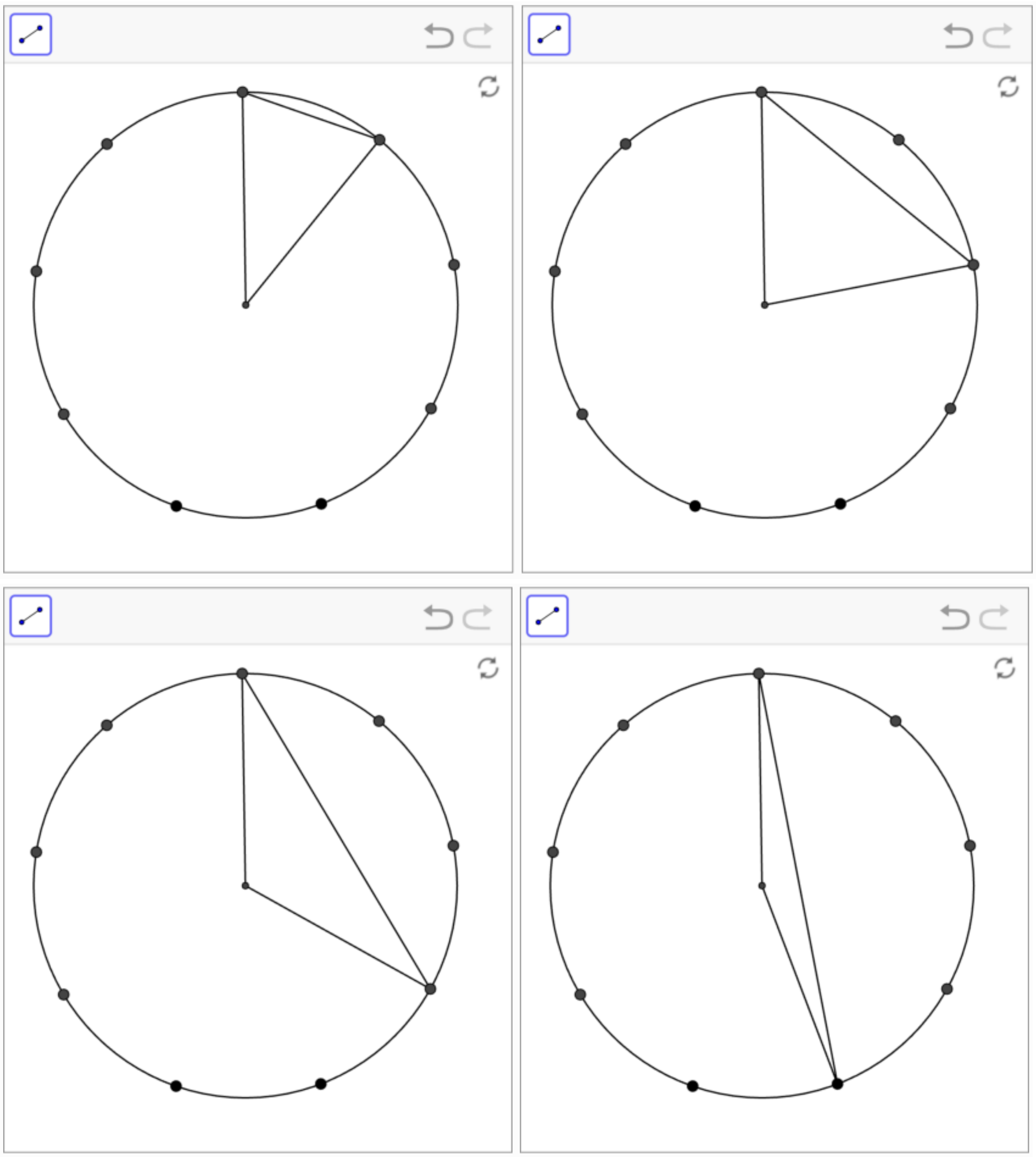
Question 1.

In the GeoGebra interactivity below there is **a circle with 9 equally spaced points on the edge**, and one in the center.

Draw as many different triangles as you can, by joining the centre dot and any two of the dots on the edge.

Answer

There are 4 different triangles that we can draw, which are:



Other triangles that one can draw are translations of one of the 4 triangles above.
For each triangle,
The angles of the first triangle are 40, 70, and 70 degrees as the inner angle is

360/(9 equally spaced vertices): 40°
The triangle is an isosceles triangle, so the edge angles are the same. Therefore,

40 + x + x = 180
x = 70

With the second triangle, the central angle is 80° because the angle is 2 central angles of the first triangle merged together. (Angle Sum Postulate)

In a similar fashion, we can calculate the edge angles, which are 50 degrees.

To repeat this process with the third and fourth triangles, we get

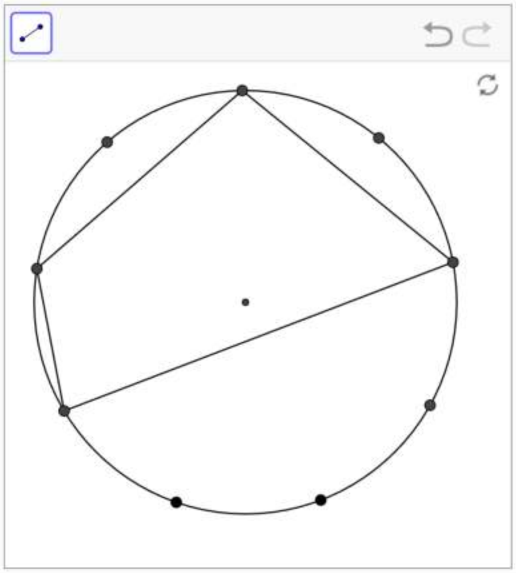
40°, 70°, 70°
80°, 50°, 50°
120°, 30°, 30°
160°, 10°, 10°

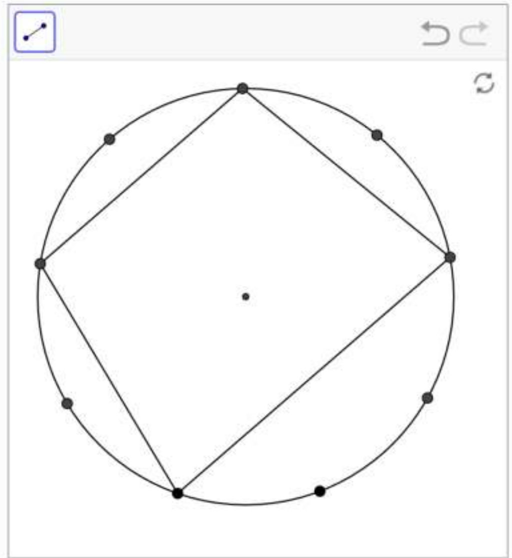
Question 2.

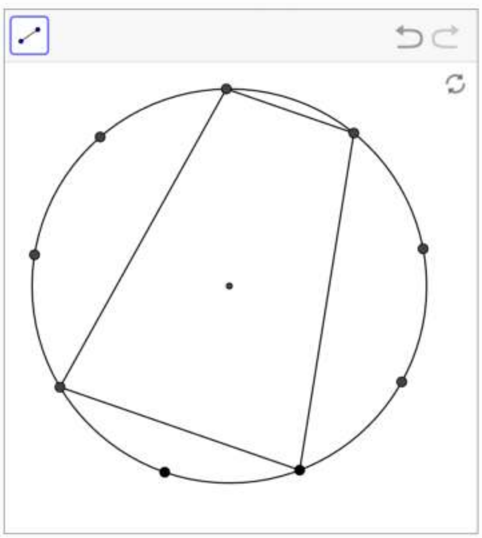
Now draw a few quadrilaterals whose interior contains the centre of the circle, by joining four dots on the edge.

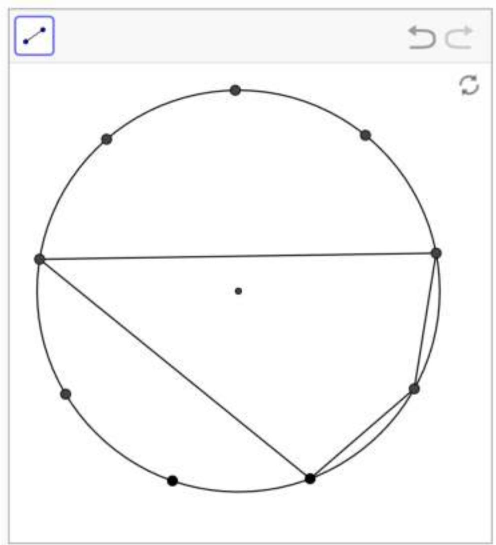
Can you work out the angles of your quadrilaterals?

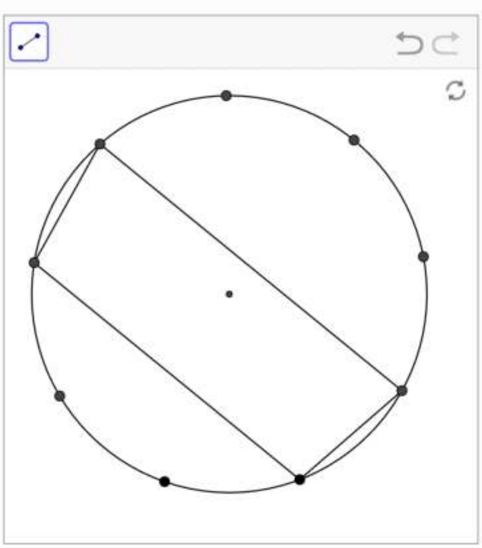
Create at least five different quadrilaterals in this way and work out their angles



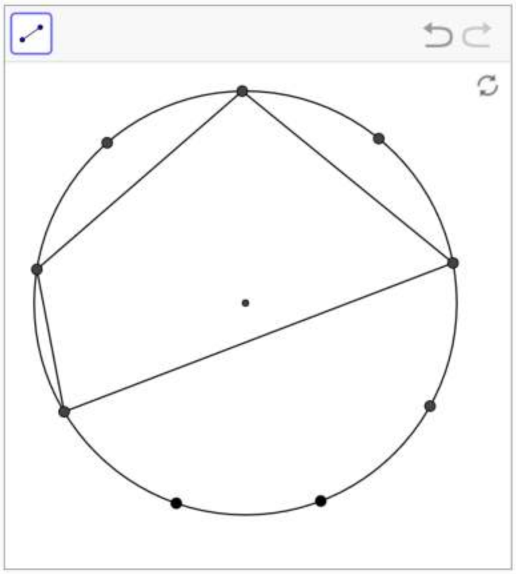




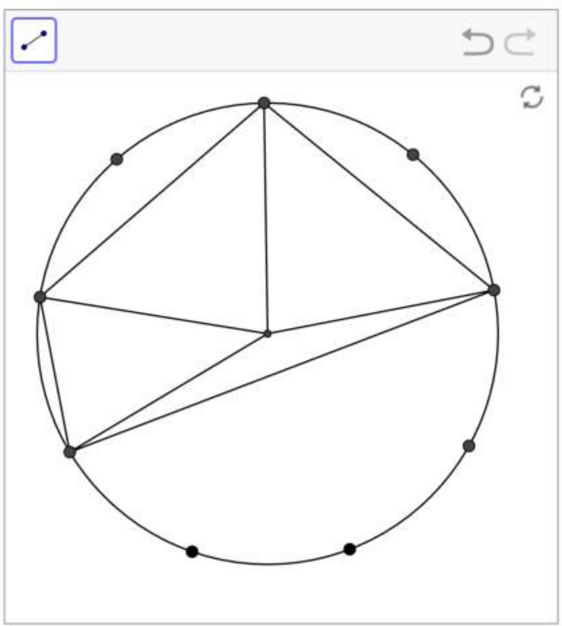




The angles of the quadrilaterals:

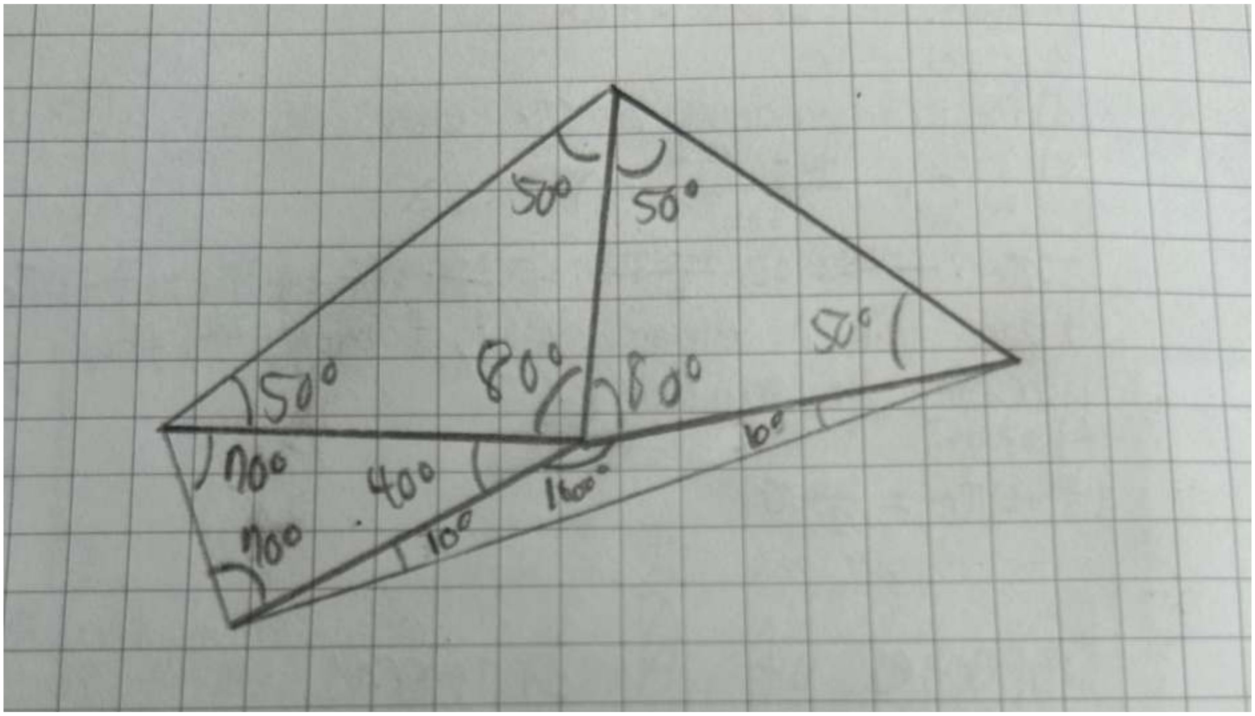


We can draw radii from the vertex of the quadrilateral to the center.

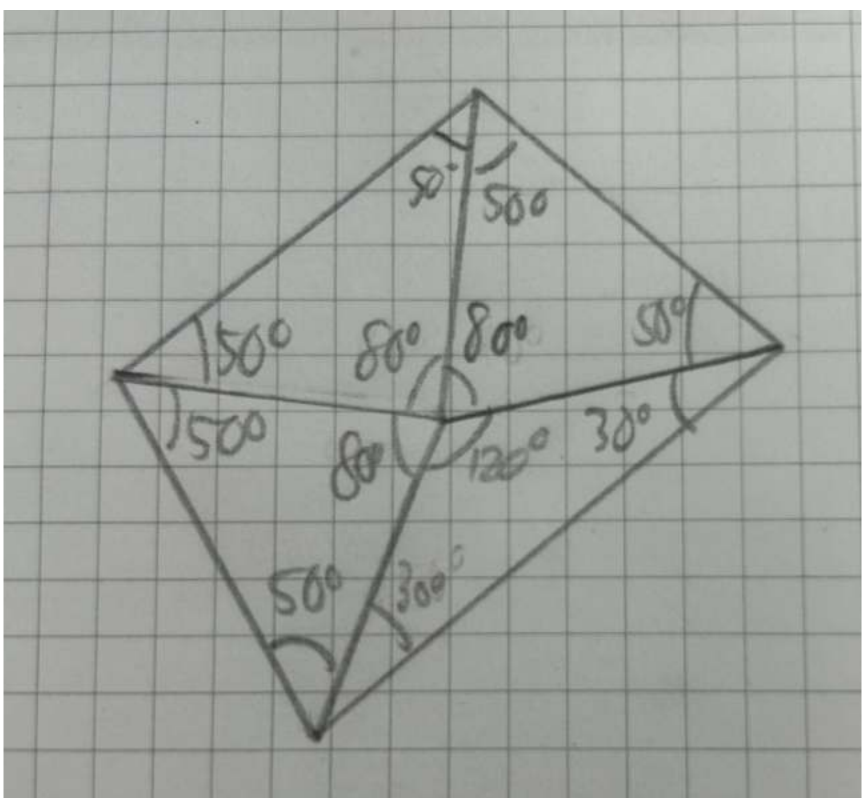


We can find the angles created from the intersections of the radii using the fact that one “slice” of the circle forms an angle of 40° because 360°/9=40°.

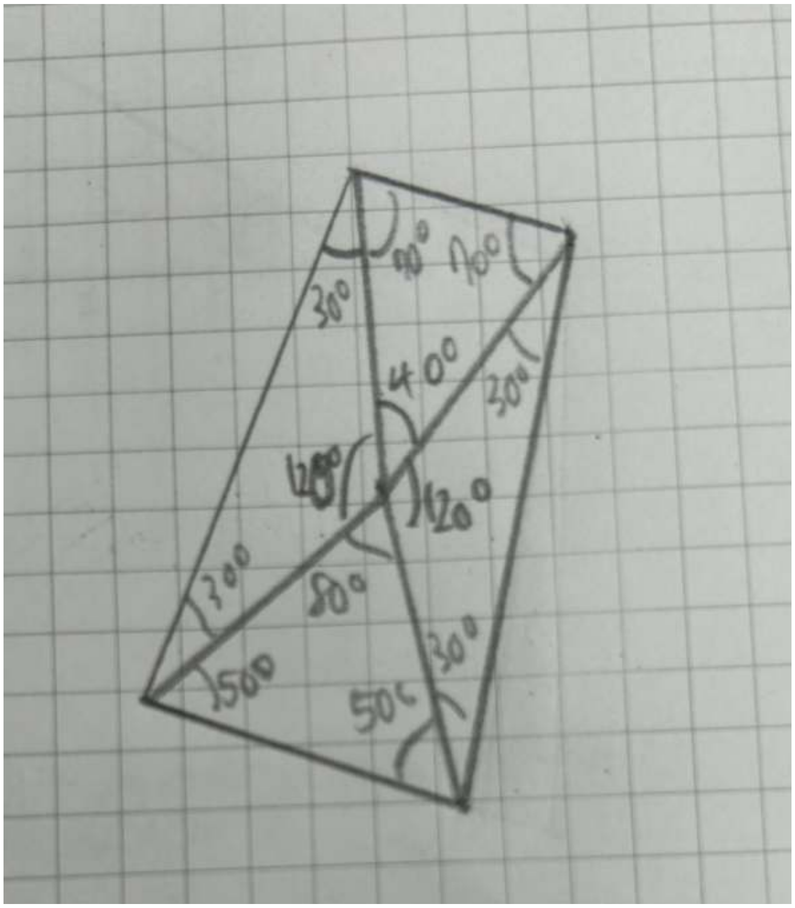
The angles formed are (from top left, top right, bottom left, bottom right) 80°, 80°, 40°, and 160°. Because all of the triangles formed are isosceles, we can find the angles of the quadrilateral.



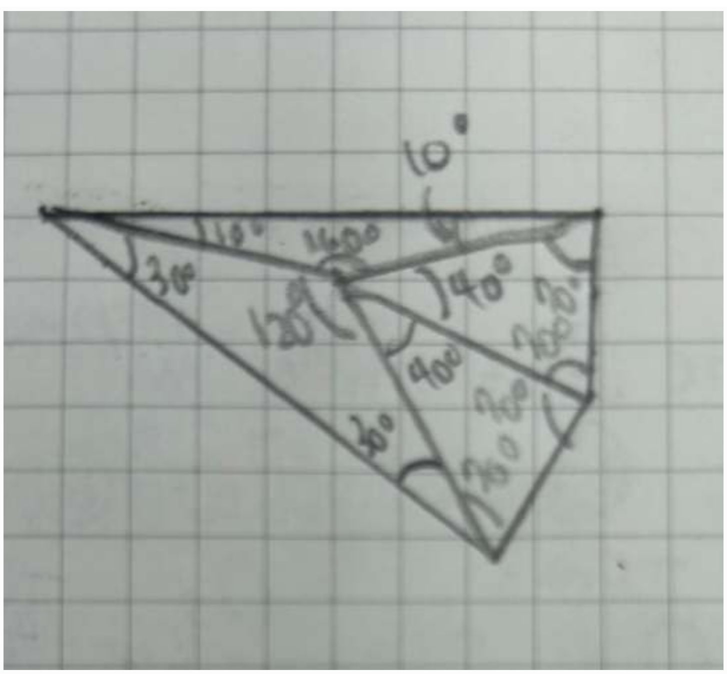
We can conclude that the angles of the quadrilateral are 120°, 100°, 80°, and 60°.
Doing the same thing for the other quadrilaterals, we get (from top left, top right, bottom left, and bottom right)



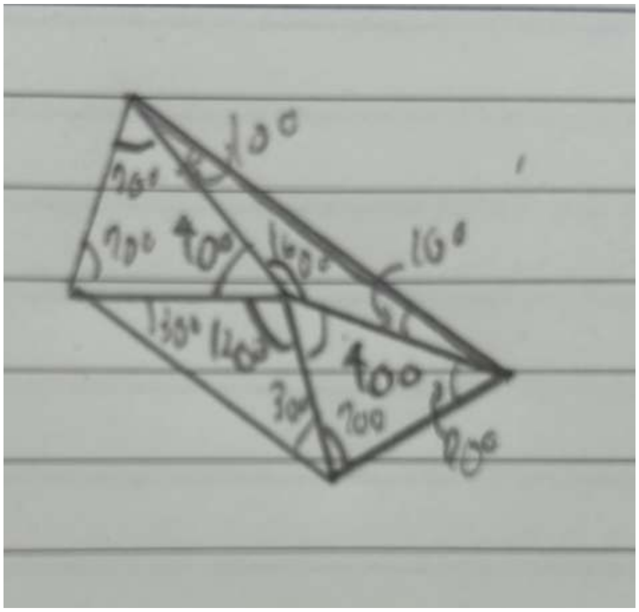
100°, 100°, 80°, and 80°



100°, 100°, 80°, and 80°



40°, 80°, 100°, and 140°



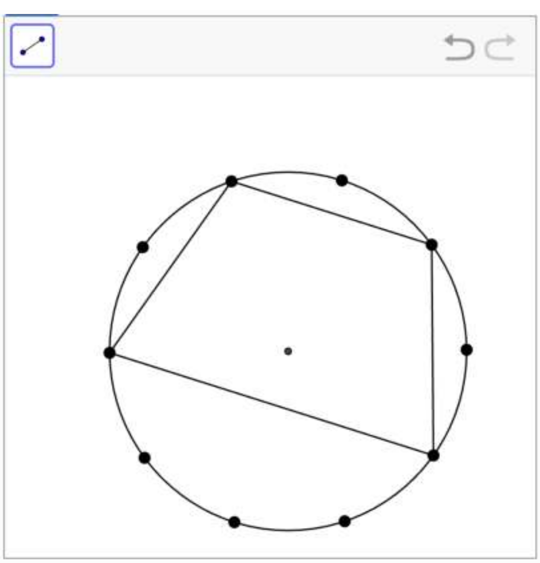
80°, 80°, 100°, and 100°

Question 3. (Justin)

Perhaps you are wondering whether this only happens with 9-dot circles...

You may wish to explore the opposite angles of quadrilaterals on circles with a different number of dots.

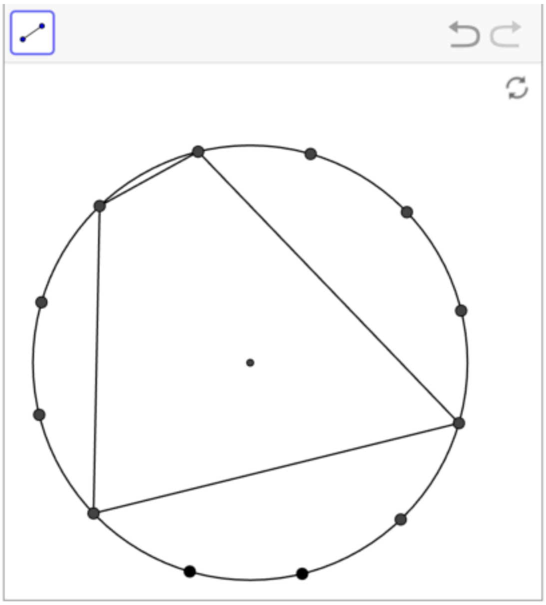
a. 10-dot circles



Using the same method as the 9-dot circles but with 36° instead of 40°

The angles of the quadrilateral are 108°, 108°, 72°, and 72°

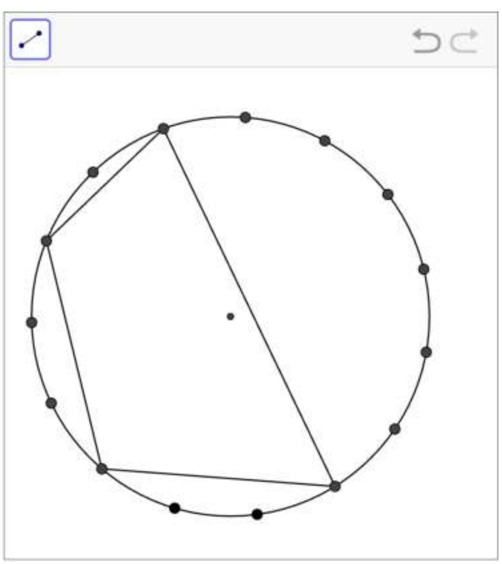
b. 12-dot circles



Doing the same thing but with 30°,

The angles of the quadrilateral are 120°, 105°, 75°, and 60°

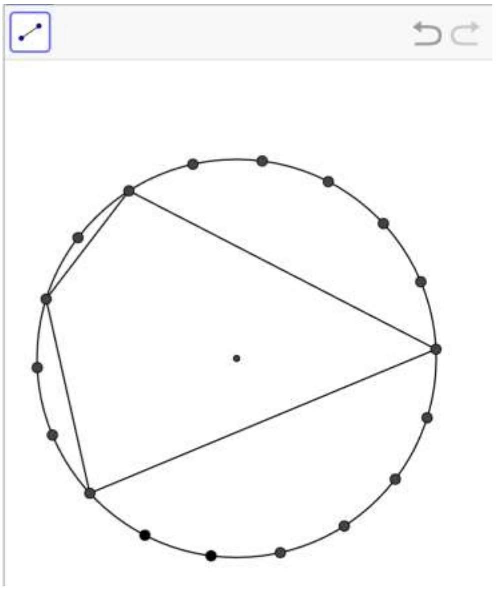
c. 15-dot circles



Doing the same thing but with 24°,

The angles of the quadrilateral are 120°, 72°, 108°, and 60°

d. 18-dot circles



Doing the same thing but with 20°,

The angles of the quadrilateral are 130°, 100°, 80°, and 50°

Question 4.

What do you notice about the angles on opposite vertices of your quadrilaterals?

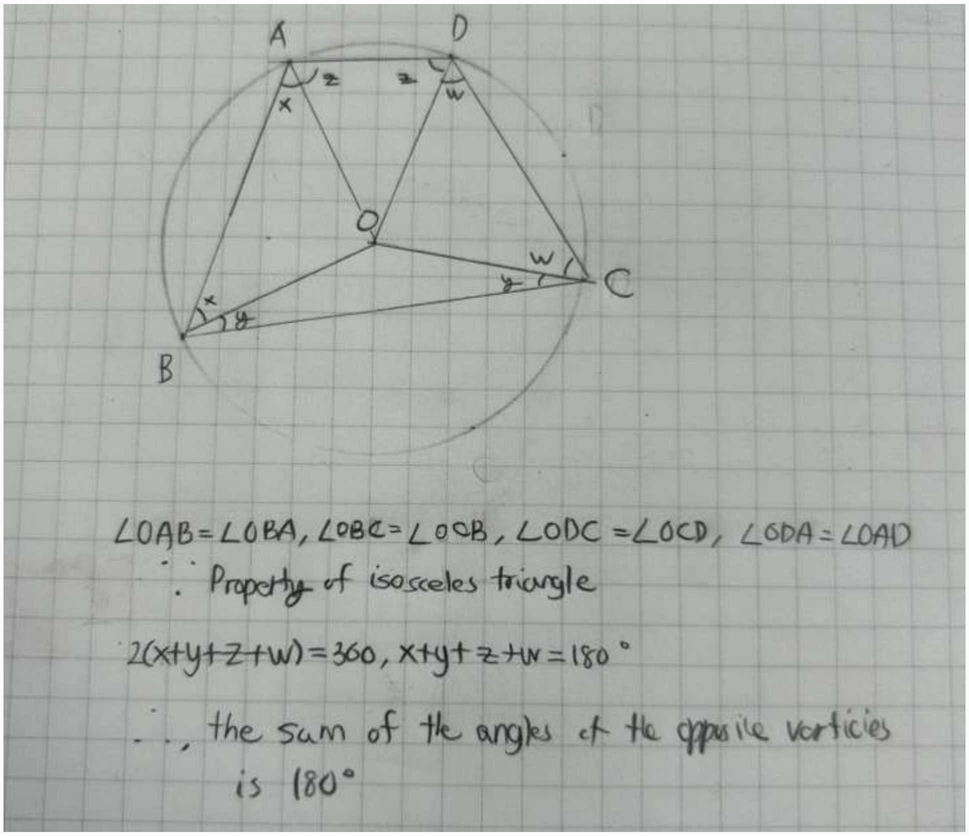
We notice that the sum of the angles on opposite sides of the quadrilaterals is 180 degrees for all of the examples provided above.

Question 5.

Extension:

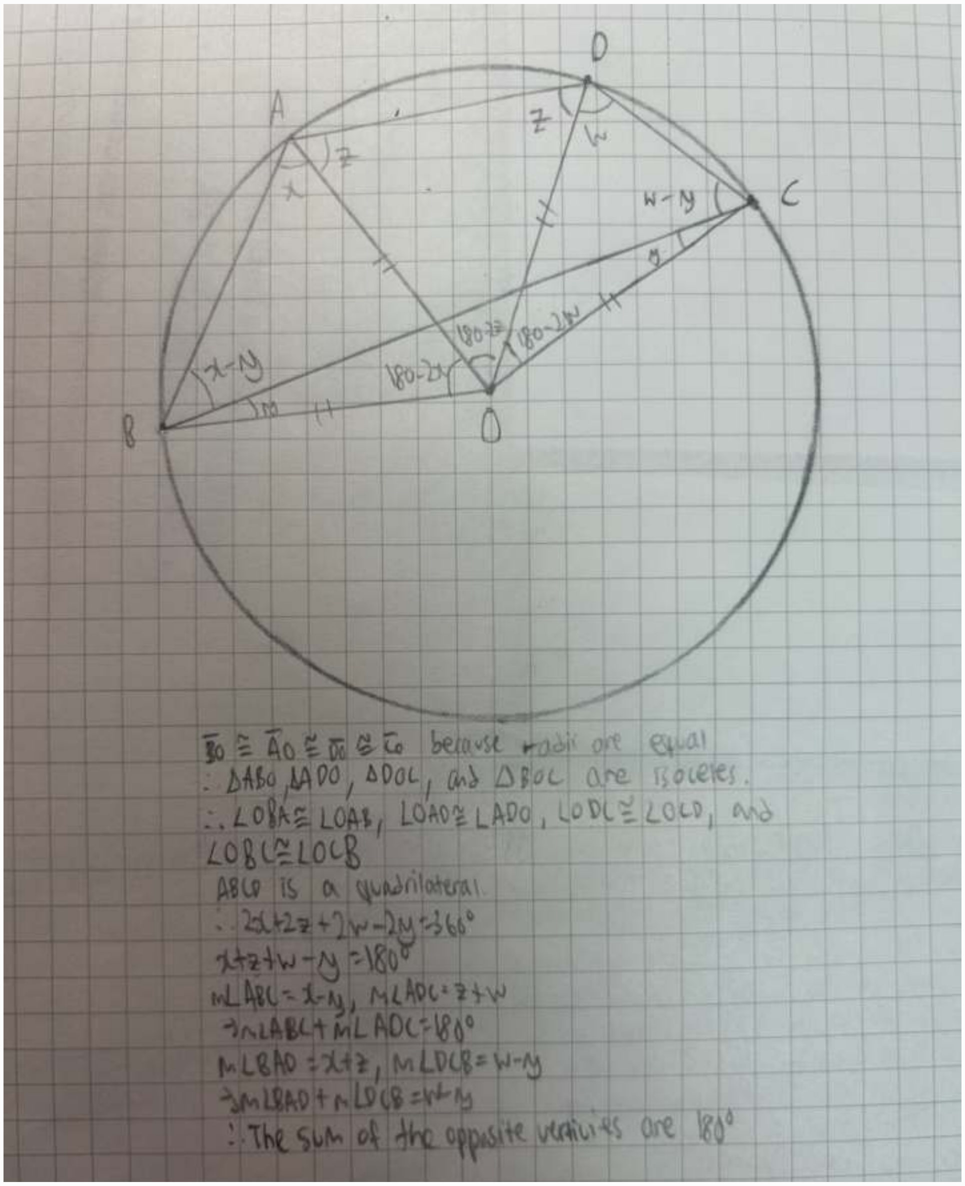
Will the same happen if you draw a circle and choose four points at random to form a quadrilateral?

a. When the center of the circle is inside the quadrilateral:



Proof by Thomas

b. When the center of the circle is outside the quadrilateral:



Proof by Justin