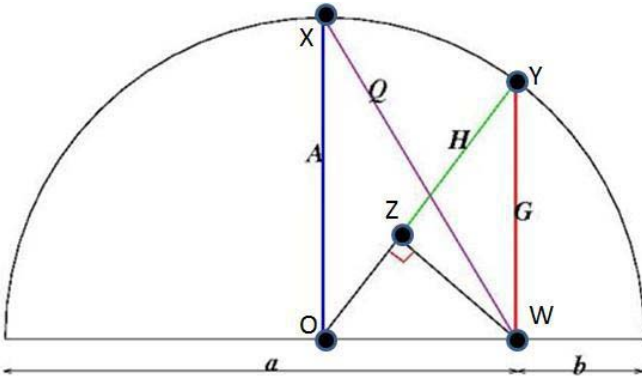


Solution by Rosa Paulina Anajao from Philippines, rosapaulina_anajao@yahoo.com

Diagram:



ii)

For A,

The diameter is $a + b$, then the radius is $\frac{a+b}{2}$. Based from the diagram, length A is also the radius of the circle. Hence, $A = \frac{a+b}{2}$

For G,

Using $\triangle OWY$, $OY = \text{radius} = \frac{a+b}{2}$ and $OW = \frac{a+b}{2} - b = \frac{a-b}{2}$

Applying Pythagorean theorem,

$$YW = G = \sqrt{(OY)^2 + (OW)^2}$$

$$G = \sqrt{\left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2}$$

By simplifying, we have: $G = \sqrt{ab}$

For H,

Since $\triangle OWY \sim \triangle WZY$

We have:

$$\frac{OY}{YW} = \frac{WY}{YZ}$$

$$\frac{\text{radius}}{G} = \frac{G}{H}$$

$$\frac{\left(\frac{a+b}{2}\right)}{\sqrt{ab}} = \frac{\sqrt{ab}}{H} \rightarrow H = \frac{2ab}{a+b} = \frac{2}{\frac{a+b}{ab}}$$

$$H = \frac{2}{\frac{1}{a} + \frac{1}{b}}$$

On the diagram, lengths A and G are both half of two parallel chords of the circle. Due to the fact that the diameter is the longest chord of the circle, it follows that $2A > 2G$. Therefore, $A > G$.

Taking ΔWZY , we can conclude that $G > H$ since G is the length of the hypotenuse.

To sum up the inequality, we have: $A > G > H$

iii) Considering ΔXOW ,

$$(XW)^2 = (XO)^2 + (OW)^2$$

$$Q^2 = A^2 + \left(\frac{a-b}{2}\right)^2$$

$$Q = \sqrt{\left(\frac{a+b}{2}\right)^2 + \left(\frac{a-b}{2}\right)^2}$$

By making the expression simpler, we would arrive to $Q = \sqrt{\frac{a^2+b^2}{2}}$