

Giants

WHAT IS BIGGER: 9^{10} OR 10^9 ?

$$9^{10} = (10 - 1)^{10}$$

Using a pyramid we can expand the brackets.

Power	Pyramid																														
0	1																														
1	1		1																												
2	1			2		1																									
3	1				3			1																							
4	1					4		6		1																					
5	1						5		10		1																				
6	1							6		15		20		15		6		1													
7	1								7		21		35		35		21		7		1										
8	1									8		28		56		70		56		28		8		1							
9	1										9		36		84		126		126		84		36		9		1				
10	1											10		45		120		210		252		210		120		45		10		1	

$$\begin{aligned}
 (10 - 1)^{10} &= 10^{10} - 10 \cdot 10^9 + 45 \cdot 10^8 - 120 \cdot 10^7 + 210 \cdot 10^6 - 252 \cdot 10^5 + 210 \cdot 10^4 - 120 \cdot 10^3 + 45 \cdot 10^2 - 10 \cdot 10^1 + 1 \\
 &= 45 \cdot 10^8 - 120 \cdot 10^7 + 210 \cdot 10^6 - 252 \cdot 10^5 + 210 \cdot 10^4 - 120 \cdot 10^3 + 44 \cdot 10^2 + 1 \\
 &= (45 - 12) \cdot 10^8 + (2,100 - 252) \cdot 10^5 + (2,100 - 120) \cdot 10^3 + 44 \cdot 10^2 + 1 \\
 &= 33 \cdot 10^8 + 1,848 \cdot 10^5 + 1,980 \cdot 10^3 + 44 \cdot 10^2 + 1 \\
 &= 3,300,000,000 + 184,800,000 + 1,980,000 + 4,400 + 1 \\
 &= 3,486,784,401 \\
 10^9 &= 1,000,000,000
 \end{aligned}$$

$$\begin{aligned}
 3,486,784,401 &> 1,000,000,000 \\
 9^{10} &> 10^9
 \end{aligned}$$

NOW FIND A WAY TO USE YOUR CALCULATOR TO COMPARE 99^{100} AND 100^{99} .

$$\begin{aligned}
 99^{100} &= (0.99 \cdot 10^2)^{100} = (0.99)^{100} \cdot (10^2)^{100} = 0.37 \cdot 10^{200} = 3.66 \cdot 10^{199} \\
 100^{99} &= (10^2)^{99} = 10^{198}
 \end{aligned}$$

$$\begin{aligned}
 3.66 \cdot 10^{199} &> 10^{198} \\
 99^{100} &> 100^{99}
 \end{aligned}$$

WORK OUT WHICH IS BIGGER OUT OF 999^{1000} AND 1000^{999} .

$$\begin{aligned}
 999^{1000} &= (0.999 \cdot 10^3)^{1000} = (0.999)^{1000} \cdot (10^3)^{1000} = 0.37 \cdot 10^{3000} = 3.68 \cdot 10^{2999} \\
 1000^{999} &= (10^3)^{999} = 10^{2997}
 \end{aligned}$$

$$\begin{aligned}
 3.68 \cdot 10^{2999} &> 10^{2997} \\
 999^{1000} &> 1000^{999}
 \end{aligned}$$