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Age Ten

Curvy Areas

The ~~area~~ radius of the circle $= r$

The Area of circle of radius is abbreviated to ACR.

Three portions -

Red: $\dfrac{ACR(r) - ACR\left(\frac{2}{3}r\right) + ACR\left(\frac{1}{3}r\right)}{2}$

$= \dfrac{1}{2}\left(\pi r^2 - \pi \dfrac{4}{9} r^2 + \pi \dfrac{1}{9} r^2\right)$

$= \dfrac{\pi r^2}{2}\left(1 - \dfrac{4}{9} + \dfrac{1}{9}\right)$

$= \dfrac{1}{3}\pi r^2$

Green: From Symettry, has the same area ~~of the~~ as the red - $\dfrac{1}{3}\pi r^2$

Orange: $\pi r^2 - \left(\dfrac{1}{3}\pi r^2 + \dfrac{1}{3}\pi r^2\right) = \dfrac{1}{3}\pi r^2$

Four portions -

Red: $\dfrac{ACR(r) - ACR\left(\frac{3}{4}r\right) + ACR\left(\frac{1}{4}r\right)}{2}$

$= \dfrac{1}{2}\left(\pi r^2 - \pi \dfrac{9}{16} r^2 + \pi \dfrac{1}{16} r^2\right)$

$= \dfrac{\pi r^2}{2}\left(1 - \dfrac{9}{16} + \dfrac{1}{16}\right)$

$= \dfrac{1}{4}\pi r^2$

Green: From symmetry same as red - $\dfrac{1}{4}\pi r^2$

Orange: $\dfrac{ACR\left(\frac{3}{4}r\right) - ACR\left(\frac{1}{2}r\right) + ACR\left(\frac{1}{2}r\right) - ACR\left(\frac{1}{4}r\right)}{2}$

$= \dfrac{1}{2}\left(\pi \dfrac{9}{16} r^2 - \pi \dfrac{1}{4} r^2 + \pi \dfrac{1}{4} r^2 - \pi \dfrac{1}{16} r^2\right)$

$$=\frac{\pi r^2}{2}\left(\frac{8}{16}\right)$$

$$=\frac{1}{4}\pi r^2$$

$\therefore$ The yellow must be $\frac{1}{4}\pi r^2$

Five portions:

Red: $\dfrac{ACR(r) - ACR\left(\frac{4}{5}\right) + ACR\left(\frac{1}{5}\right)}{2}$

$$=\frac{1}{2}\left(\pi r^2 - \frac{16}{25}\pi r^2 + \frac{1}{25}\pi r^2\right)$$

$$=\frac{\pi r^2}{2}\left(1 - \frac{16}{25} + \frac{1}{25}\right)$$

$$=\frac{1}{5}\pi r^2$$

Blue: Is symmetrical to red $\Rightarrow$ area $=\frac{1}{5}\pi r^2$

Orange: $\dfrac{ACR\left(\frac{4}{5}r\right) - ACR\left(\frac{3}{5}r\right) + ACR\left(\frac{2}{5}r\right) - ACR\left(\frac{1}{5}r\right)}{2}$

$$=\left(\frac{16}{25}\pi r^2 - \frac{9}{25}\pi r^2 + \frac{4}{25}\pi r^2 - \frac{1}{25}\pi r^2\right)\frac{1}{2}$$

$$=\frac{\pi r^2}{2}\left(\cancel{1}\ \frac{16}{25} - \frac{9}{25} + \frac{4}{25} - \frac{1}{25}\right)$$

$$=\frac{1}{5}\pi r^2$$

Generalisation:

For n portions, the area of each portion $=$ ~~1/n~~ $\dfrac{\pi r^2}{n}$

Proof:

If I can express any of the portions to have an area of $\frac{1}{k}$ - where k is the ~~tot~~ total number of portions - I will prove that my generalisation is correct.

The area of the portion that is $i-1$ portions away from the edge = ~~area of~~ cummulative sum of areas until that portion + cumulative sum of areas until one before that portion

$$=\left(\pi r^2-\left(\frac{k-i}{k}\right)^2\pi r^2+\frac{i^2}{k^2}\pi r^2\right)\frac{1}{2}-\left(\pi r^2-\left(\frac{k-i+1}{k}\right)^2\pi r^2+\frac{i^2}{k^2}\pi r^2\right)\frac{1}{2}$$

$$\cancel{=\frac{(k-i)-(k-i)^2+(k-(i+1))^2+2i^2}{k^2}\frac{\pi r^2}{2}}$$

$$=\frac{\pi r^2}{2}\left(\frac{(k-i+1)^2-(k-i)^2+i^2-(i-1)^2}{k^2}\right)$$

$$=\frac{\pi r^2}{2}\left(\frac{(2k-2i+1)+(2i-1)}{k^2}\right)$$

$$=\frac{\pi r^2}{2}\,\frac{2k}{k^2}$$

$$=\frac{\pi r^2}{k}$$

This shows that the area of one portion when the circle is divided onto k parts $=\pi r^2 \cdot \frac{1}{k}$

$\therefore$ Every part has the same area and my generalisation is proved.