

Symmetry

when we add the five equations:

$$\begin{aligned}(1): & b+c+d+e=4 \\(2): & a+c+d+e=5 \\(3): & a+b+d+e=1 \\(4): & a+b+c+e=2 \\(5): & a+b+c+d=0\end{aligned}$$

we obtain:

$$(6): 4(a+b+c+d+e)=12$$

dividing by 4 we get:

$$(7): a+b+c+d+e=3$$

as we subtract (4), (3), (4) and (5) from (7) we get

$$\begin{aligned}8): & a = -1 \\9): & b = -2 \\10): & c = 2 \\11): & d = 1 \\12): & e = 3\end{aligned}$$

As per multi:

when we multiply the three equations:

$$\begin{aligned}(1): & xy = 4 \\(2): & yz = 4 \\(3): & zx = 9\end{aligned}$$

we obtain:

$$(4): x^2y^2z^2 = 36$$

as we square root we obtain:

$$(5): xyz = \pm 6$$

as we divide (5) by (1), (2) and (3) we get

$$\begin{aligned}(6): & z = \pm 6 \\(7): & x = \pm 1.5 \\(8): & y = \pm 2/3\end{aligned}$$

where x, y and z are all positive or all negative  
when we multiply the 5 equations:

$$\begin{aligned}(1): & ab = 1 \\(2): & bc = 2 \\(3): & cd = 3 \\(4): & de = 4 \\(5): & ea = 5\end{aligned}$$

we obtain:

$$(6): a^2b^2c^2d^2e^2 = 144$$

as we square root we obtain:

$$(7): abcde = \pm 12$$

if we divide by (7) by (1) and (3); (2) and (4); (3) and (5); (4) and (1); and (5) and (2) we get

$$(8): a = \pm \frac{2}{5} \pm \frac{3}{2}$$

$$(9): b = \pm \frac{2}{5} \pm \frac{3}{2}$$

$$(10): c = 3^3$$

$$(11): d = 1$$

$$(12): e = \pm 4$$

where  $a, b, c, d$  and  $e$  are all positive or all negative