Nrich – Integration Master

(1) It can be observed that Chart J is of the form $y=a$ (it is $y=1$), where 'a' is a constant. Therefore, when integrating the function $f(x)=a$, we get

$$\int f(x).dx = \int a.dx$$

$$= ax + c$$

Chart A represents a function of this form ($y=mx+c$) where c is equal to zero, ~~therefore~~ therefore making the equation for chart A $y=mx$.

Therefore, we can see that the first pair of charts is chart J and chart A, ~~whic~~ with the former integrating to become the latter.

(2) Chart D can be seen to represent a cubic function, therefore taking the form $ax^3+bx^2+cx+d=0$. Therefore we ~~g~~ can integrate this cubic function to get

$$\int f(x)\,dx = \int ax^3+bx^2+cx+d\ \ dx$$

$$= \frac{ax^4}{4}+\frac{bx^3}{3}+\frac{cx^2}{2}+dx+e$$

Since e is an arbitrary constant, we can ~~get~~ set e to equal zero, therefore obtaining a curve that passes through the origin. This new function $\frac{ax^4}{4}+\frac{bx^3}{3}+\frac{cx^2}{2}+dx=0$ represents the form of a biquadratic ~~equation~~ function, which is represented in Chart B.

Therefore we can integrate Chart D to get Chart B

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③ Chart K represents the function $y = \cos x$. This function can be integration to get

$$\int f(x).dx = \int \cos x \; dx$$

$$= \sin x + c$$

Chart F represents a function $y = \sin x + c$, where $c = 0$, giving the function $y = \sin x$.

Therefore, we can see the function represented in Chart K can be integrated to give the function represented in Chart F

④ In Chart I, ~~&~~ the function can be observed with respect to the fact that the rate at which the area under the graph is increasing, is decreasing. There are only 2 charts which show that the rate ~~as~~ at which the area is increasing is continuously decreasing are charts C and H. However, chart C implies that some of the area of the function in the given range is negative. This is not the ~~et~~ case as in chart I $f(x) > 0$.

Therefore, Chart I can be integrated to get Chart H.

⑤ In Chart G, the area under the curve between intervals is equal. The first interval is from 0 to approximately 3.1 and then the next interval is shown to be from ~~near~~ approximately 3.1 to 6.3, and this continues. Furthermore, the rate at which the area increases in the intervals first increases, then decreases.

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cont. ⑤ Chart E is the only chart that displays such a trend. And as seen in both Charts G and Charts E, this sequence repeats.

Therefore, we can see that the function in Chart G integrates to the function in Chart E.

⑥ Finally, this leaves just Charts C and L. For Chart C, integrating the function from $x=1$ to $x=4$ will give a negative area.

In Chart C, let $g(x)$ be the integral of the function $f(x)$ in the chart. Chart L ~~clearly~~ shows that the magnitude of $g(x)$ increases from $x=1$ to $x=4$. As expected from Chart C, the area becomes more negative. Then, from $x=4$ ~~to~~ onwards, positive area is added to the negative area, increasing the value of the total ~~one~~ area. This shows that as $|g(x)|$ (the absolute ~~value~~ value of the function $g(x)$) decreases, $g(x)$ ~~bec~~ decreases.

Therefore, this clearly denotes that Chart C integrates to become the function in Chart L.