

## TASK A

1. If  $x$  is odd, then the formula  $3^x + 2^x$  will always end in 5 (and therefore be part of the sequence  $10n + 5$ ).

1. Explanation: At first, the basics must be thought about;  $2 + 3 = 5$ . From this, you can build up your explanation.  $2^2 + 3^2$ , or  $2^x + 3^x$  for any even value of  $x$ , for that matter, will not end in 5. However, I noticed that these even powers would end successively in 3, 7, 3, 7, etc. Interestingly, both of these numbers (7 & 3) differentiate from 5 by 2. So, why all the odd ones? 3 to the power of an odd number always ends in (once again) 3 and 7: 3, 27, 243, 2187, 19683, etc. Secondly, 2 to the power of an odd number will end in 2 or 8: 2, 8, 32, 128, 512, etc. Because  $2 + 3$  is 5, and  $7 + 8$  ends in 5, it now seems obvious why  $2^x + 3^x$  (with an odd value of  $x$ ) will end in 5. This answers the second question, as I have already stated that when  $x$  is a multiple of 4,  $2^x + 3^x$  will end in 7.

2. All of them. This is because 1 to the power of any number is 1, an odd number. 3 to the power of any number will also be odd. 2 to the power of any number will be even (a multiple of 2). Odd + Odd = Even, and Even + Even = Even. Therefore  $1^n + 2^n + 3^n$  will be even, for any value of  $n$ .

3. The sequence will go like this (in terms of the last digit of the numbers): 0, 0, 0, 4, 0, 0, 4, etc.

4. The end digit of the numbers will go like this: 5, 5, 5, 9, 5, 5, 5, 9, 5, etc. This is easy to explain, because of the answer to Q3.  $5^n$ ,  $n$  being an abstract number, will end in 5. Taking into account the last question, add 5 to the power of last digit of each number, and you get the sequence of last digits I mentioned earlier (5, 5, 5, 9, etc.).

Other:

1. ( $4^n + 5^n + 6^n$ ): In this sequence, the numbers will end like this: 5, 77, 5, 77, etc.  $4^n$  will end in these digits: 4, 6, 4, 6, 4, 6, etc.  $5^n$  will end in 5 and  $6^n$  will end in 6, so it is no surprise that the last digits of the added numbers will be 5, 7, 5, 7, etc. However, I am unsure why the last two digits of the number (for an even value of  $n$ ) will be 77.

2. ( $3^n + 8^n$ ): In this sequence, the numbers will end like this: 11, 73, 39, 77, 11, 39, etc.  $8^n$  ends 8, 4, 2, 6, 8, 4, 2, 6, etc.  $3^n$  ends 3, 9, 7, 1, 3, 9, 7, 1, etc. Add these together and you get 1, 3, 9, 7, 1, 3, 9, 7, etc. Again, I am not sure where the second last digit comes in.

3. ( $2^n + 4^n + 6^n$ ): Here, the last digits of the numbers will be 2, 6, 8, 8, 2, 6, 8, 8, etc.  $2^n$  ends 2, 4, 8, 6, 2, 4, 8, etc.  $4^n$  ends 4, 6, 4, 6, etc.  $6^n$  ends in 6. Add these together to get last digits of 2, 6, 8, 8, 2, 6, 8, 8, etc.

4. ( $3^n + 5^n + 7^n$ ): In this sequence, the numbers will end: 5, 3, 5, 7, 5, 3, 5, 7, etc.  $3^n$  ends 3, 9, 7, 1, etc.  $7^n$  ends 7, 9, 3, 1, 7, 9, 3, 1, etc.  $5^n$  ends 5. Add these together to get 5, 3, 5, 7, 5, 3, 5, 7, etc.

5. ( $3^n - 2^n$ ): This time, the numbers end in these digits: 1, 5, 9, 5, 1, 5, 9, 5, etc.

$3^n$  ends 3, 9, 7, 1, 3, 9, 7, 1, etc.  $2^n$  ends 2, 4, 8, 6, 2, 4, 8, 6, etc. subtract the latter from the former to get 1, 5, 9, 5, 1, 5, 9, 5, etc.

6.  $(7^n + 5^n - 3^n)$ : In this sequence, the numbers end like this: 9, 5, 1, 5, 9, 5, 1, 5, etc.  
 $7^n$  ends in 7, 9, 3, 1, 7, 9, 3, 1, etc.  $5^n$  ends in 5.  $3^n$  ends 3, 9, 7, 1, 3, 9, 7, 1, etc.  $(7,9,3,1) + 5 - (3,9,7,1) = (1,5,9,5)$ .

## TASK B

1. What is half of  $2^{20}$ ?

- A  $1^{10}$
- B  $1^{20}$
- C 20
- D  $2^{10}$
- E  $2^{19}$**

2. Which of these numbers is biggest?

$\times 99$   
 $\times 9$   
 **$199^9$**   
 $9^9$

- A 19
- B 199
- C**
- D  $1^9 \times$
- E  $1^{999}$

3. Below are three statements. Exactly which ones are true?

(i)  $3^{10}$  is even                      (ii)  $3^{10}$  is odd                      (iii)  $3^{10}$  is square

only  
 only  
 only  
 and (iii)  
**and (iii)**

- A (i)
- B (ii)
- C (iii)
- D (i)
- E (ii)**

4. The number  $3^4 \times 4^5 \times 5^6$  is written out in full.

How many zeros are there at the end of the number?

**none**

- A**
- B 4

- C 5
- D 6
- E

more than 6

5. Which of these five expressions represents the largest number?

$$9^{(9^9)}$$

- A**
- B 999
- C  $9^{99}$
- D  $(9^9)^9$
- E  $99^9$