



Let x be the side length of the squares being cut out.

Let y be the side length of the whole sheet of paper.

$$\text{Volume } V = (y - 2x)^2 x = 4x^3 - 4yx^2 + y^2 x$$

$$\frac{dV}{dx} = 12x^2 - 8yx + y^2 = (6x - y)(2x - y)$$

At turning points $\frac{dV}{dx} = 0$, so $x = \frac{y}{6}$, $x = \frac{y}{2}$

At maximum $\frac{d^2V}{dx^2} < 0$

$$\frac{d^2V}{dx^2} = 24x - 8y$$

$$24\left(\frac{y}{6}\right) - 8y = -4y \text{ - this is the maximum}$$

$$24\left(\frac{y}{2}\right) - 8y = 4y$$

$$\text{At maximum } V, x = \frac{y}{6}, V = \left(y - 2\left(\frac{y}{6}\right)\right)^2 \times \frac{y}{6}$$

$$= \frac{2y^3}{27}$$

Here $y = 20 \text{ cm}$

$$x = \frac{y}{6} = \frac{10}{3} \text{ cm}$$

$$V = \frac{2(20)^3}{27} = \frac{16000}{27} \text{ cm}^3$$