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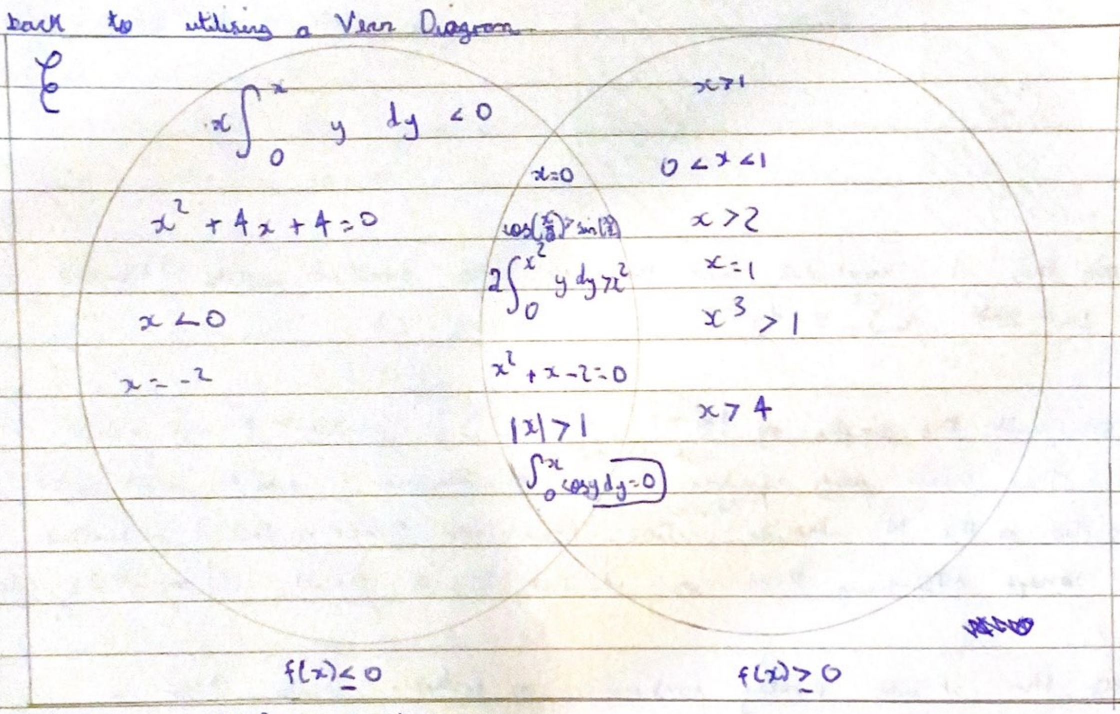
Note: My ⇒ and ⇒ signs look similar. Only when the = is connected to > is it if ... then...

Mind your Ps and Qs

When I first encountered this problem, I was boggled by the '⇒' and the '⇔' sign. However some use of the internet helped me to solve this problem and understand the importance of these signs not just in this problem, but in mathematics in general.

After this, I primarily focused on sorting the 'propositions' into two distinct sets with a crossover not dissimilar to a Venn Diagram. However, I tried to sort the 'propositions' into groups without the use of a Venn Diagram. This worked but wasn't useful visually. As a result, I retreated back to utilising a Venn Diagram.



x - Domain   f(x) = Range of values

Following on from this it was evident that some of these were able to be separated. The two polynomials: $x^2+x-2=0$ and $x^2+4x+4=0$

By using factorisation by inspection methods:

➔ $x^2 + x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0$

$\Rightarrow x = -2$ or $x = 1$

➔ $x^2 + 4x + 4 = 0$

$\Rightarrow (x+2)^2 = 0$

$\Rightarrow x = -2$

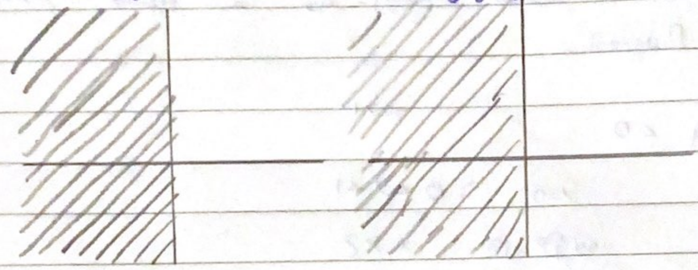
From here, two sets of statements could be matched up:

$x^2 + 4x + 4 = 0 \Leftrightarrow x = -2$

$x^2 + x - 2 = 0 \Leftrightarrow x = 1$

Hereafter, I used the 'table' function on my calculator along with my knowledge of quadratic/linear inequalities and my knowledge on integrals to plot 'sketches' of each of the remaining graphs. From them, two popped out as blatantly identical.

$x < 0$ $\qquad$ $x\int_0^x y \, dy < 0$



(Note all graphs extend onwards to infinity in all axes).

From this, I concluded that this should be another match. Therefore

$x < 0 \Leftrightarrow x\int_0^x y \, dy < 0$

Again with the graphs of $|x| > 1$ and $2\int_0^{x^2} y \, dy > x^2$ I noticed that they were very similar, but not identical. This was because I didn't release of the function of the '$|x|$' absolute/modulus function. Again after research on this I understood the concept determining them as identical. As a result, $|x| > 1 \Leftrightarrow 2\int_0^{x^2} y \, dy > x^2$

After this it was purely combinations of probable choices. After a while, I worked out:

$x > 1 \Rightarrow x^3 > 1$

and

$x > 4 \Rightarrow x > 2^3$.

Finally, I needed to ~~learn~~ revisit/remind myself of the concept of radians to understand the rest. The graph of $\cos(\frac{x}{2}) > \sin(\frac{x}{2})$ went from ~~[illegible]~~ -270 degrees to 90. Conversion to radians showed, -1.5π to 0.5π radians which repeated every 2π radians. This meant it had a value between 0 and 1. Therefore if x is between 0 and 1 then it fits the criteria of $\cos(\frac{x}{2}) > \sin(\frac{x}{2})$.

Therefore, I concluded:

$$0 < x < 1 \Rightarrow \cos\left(\frac{x}{2}\right) > \sin\left(\frac{x}{2}\right).$$

This left $x = 0$ and $\int_0^x \cos y \, dy = 0$. As I learnt before, the derivate of sin y is cos y. So when integrating cos y, it would be sin y. As a result I hypothesised lines at the points of when a sin y graph would cross the x-axis. And lo and behold, I was correct. As a result, I concluded that since sin y crosses the axes at 0,0, there should be a line present. Therefore if $x = 0$, $\int_0^x \cos y \, dy = 0$. As a result, $x = 0 \Rightarrow \int_0^x \cos y \, dy = 0$.

To conclude, this was an interesting puzzle in which I learnt many new concepts. Below are my final answers:

$$x^2 + 4x + 4 = 0 \Leftrightarrow x = -2$$
$$x^2 + x - 2 = 0 \Leftrightarrow x = 1$$
$$x > 1 \Rightarrow x^3 > 1$$
$$x > 4 \Rightarrow x > 2$$
$$|x| > 1 \Leftrightarrow 2\int_0^{x^2} y \, dy > x^2$$
$$x < 0 \Leftrightarrow x\int_0^x y \, dy < 0$$
$$0 < x < 1 \Rightarrow \cos\left(\frac{x}{2}\right) > \sin\left(\frac{2}{2}\right)$$
$$x = 0 \Rightarrow \int_0^x \cos y \, dy = 0$$