Searching for Mean(ing)

Mahdi Raza Khunt
Mahatma Gandhi International School

Sunday, 2 February 2020

First we define algebraically what mean is, a and b are the two rods. x and y are the number of times we use a and b respectively

$$\text{Mean} = \frac{x(a) + y(b)}{x + y}$$

Combinations of means for 3 and 8. They can give any mean weight

$$\text{Mean} = \frac{x(3) + y(8)}{x + y}$$

Smallest mean = 3
Largest mean = 8

$$3 = \frac{5(3) + 0(8)}{5 + 0} \implies 3$$

$$4 = \frac{4(3) + 1(8)}{4 + 1} \implies 4$$

$$5 = \frac{3(3) + 2(8)}{3 + 2} \implies 5$$

$$6 = \frac{2(3) + 3(8)}{2 + 3} \implies 6$$

$$7 = \frac{1(3) + 4(8)}{1 + 4} \implies 7$$

$$8 = \frac{0(3) + 5(8)}{0 + 5} \implies 8$$

Rods with difference 2

We first try to make mean with lengths of difference 2

Let $a, a+2$ be the two lengths with difference two.

$$\text{Mean} = \frac{x(a) + y(a+2)}{x+y}$$

The form to find the mean for this case is the equation given above. We will use it throughout to derive our solution

When $y = 0$, Regardless of the values of x and a we get

$$= \frac{x(a) + 0(a+2)}{x+0}$$
$$= \frac{x(a)}{x}$$
$$= (a)$$

When $x = 0$, Regardless of the values of y and a we get

$$= \frac{0(a) + y(a+2)}{0+y}$$
$$= \frac{y(a+2)}{y}$$
$$= (a+2)$$

When $x = y$, Regardless of the value of a we get

$$= \frac{x(a) + x(a+2)}{x+x}$$
$$= \frac{x(2a+2)}{2x}$$
$$= \frac{2x(a+1)}{2x}$$
$$= (a+1)$$

We have shown that we can make any mean between the length of rods if the difference between the two rods is 2. Also the smallest mean is a and the largest mean is $a+2$

Rods with difference 'd'

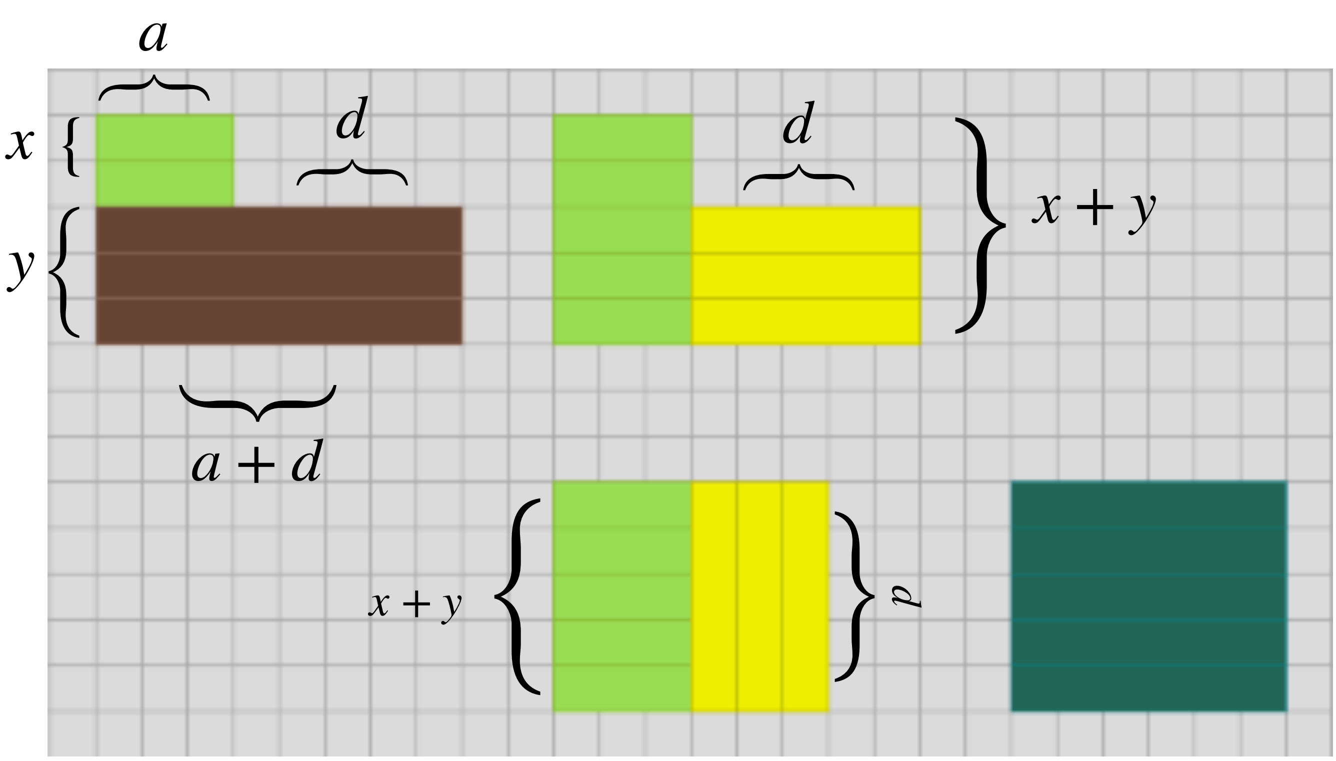
Let $a, a+d$ be the two lengths with difference 'd'.

$$\text{Mean} = \frac{x(a) + y(a+d)}{x+y}$$

$$\text{Mean} = \frac{(x+y)a + yd}{(x+y)}$$

$$\text{Mean} = \frac{(x+y)a}{(x+y)} + \frac{yd}{(x+y)}$$

$$\text{Mean} = a + \frac{yd}{(x+y)}$$



From the graphic, we can see $d = x + y$.

Back to the equation,

$$\text{Mean} = a + \frac{yd}{(x + y)}$$

$$d = x + y \implies \text{Mean} = a + \frac{yd}{d}$$

$$\boxed{\text{Mean} = a + y}$$

We know the value of a and the mean we want to calculate, so we can obtain y. After we know y, since we already know d we can find x and we are done

Example: Let the rods be 2 and 7 with $d = 5$,

To find the mean of 6,

$$\text{Mean} = a + y$$

$$6 = 2 + y \implies y = 4 \implies x = 1 \quad (\because d = 5)$$

$$\text{Mean} = \frac{x(a) + y(a + d)}{x + y}$$

$$\text{Mean} = \frac{1(2) + 4(7)}{4 + 1} = \frac{30}{5} \equiv 6$$

Similarly, we can get all means:-

$$\boxed{a, \quad a + 1, \quad a + 2, \quad a + 3, \quad \ldots \quad a + (d - 1), \quad a + d}$$

General algorithm:

1. Find d, the difference between the two rods.
2. To get a mean $a+m$, where $0 \leq m \leq d$, use $(d-m)$ such rods for a and m such rods for $(a+d)$

Proof of algorithm:

$$\text{Mean} = \frac{x(a) + y(a+d)}{x+y}$$

$$a + m = \frac{(d-m)(a) + (m)(a+d)}{(d-m) + (m)}$$

$$a + m = \frac{ad - am + am + md}{d}$$

$$a + m = \frac{ad \cancel{- am + am} + md}{d}$$

$$a + m = \frac{d(a+m)}{d}$$

$$\boxed{a + m \equiv a + m}$$

Examples questions from the problem:

Back to the equation, we have to find mean for 17 and 57.

By the algorithm to find the mean of 44 , we have to first find difference d and then m

$$\underline{d = |57 - 17| = 40}$$

$$44 = a + m$$
$$m = 44 - 17 = 27$$

$$44 = \frac{13(17) + 27(57)}{40}$$
$$44 \equiv 44$$

Similarly, we can find mean for 52 and 21 respectively.

$$52 = a + m$$

$$m = 52 - 17 = 35$$

$$52 = \frac{5(17) + 35(57)}{40}$$

$$52 = \frac{2080}{40}$$

$$52 \equiv 52$$

$$21 = a + m$$

$$m = 21 - 17 = 4$$

$$21 = \frac{36(17) + 4(57)}{40}$$

$$21 = \frac{840}{40}$$

$$21 \equiv 21$$

∎