M, M, and M Solution

In this problem, I was supplied with 4 pieces of information: the mean, median, mode, and the count (5) of several sets of positive whole numbers.

The mean of each set is 4. This tells me that all five numbers in the set should add up to 20 (mean * count of each set).

Because 3 is the median, you can fill in one blank right away! A set should look something like this:

_, _, 3, _, _

Also, I know that the mode is 3. Hence, I can fill in more than one blank with 3. So, the number 3 can occur 4, 3, or 2 times in a set. Now comes the tricky part, finding out how many sets are possible.

Case I:

Consider the simplest case. Suppose 3 occurs four times in a set. Given that 3*4= 12, I subtract 12 from 20 to get the remaining number. Hence, 20-12=8.

Therefore, the set of numbers is 3, 3, 3, 3, 8.

Case II:

Suppose 3 occurs thrice.

3*3= 9

So, the sum of the remaining two numbers is 20-9=11

To find the remaining two numbers, I kept in mind the following:

1. Numbers can be repeated twice.
2. The two numbers should add up to 11.

There are 6 possible ways to make up the number 11 using 2 positive whole numbers:

(0+11), (1+10), (2+9), (3+8) (4+7), and (5+6).

The set 3, 3, 3, 3, 8 is already covered in Case I, so that leaves me with 5 possible sets.

Case III:

Suppose 3 occurs twice.

3*2= 6

So, the sum of the remaining three numbers is 20-6=14

To find the remaining three numbers, I kept in mind the following:

1. There should be no repeating numbers because 3, which is the mode, repeats twice.
2. The three numbers should add up to 14.

There are 16 possible ways to make up the number 14 using 3 positive whole numbers:

(0+1+13), (0+2+12), (0+3+11), (0+4+10) (0+5+9), (0+6+8), (1+2+11), (1+3+10), (1+4+9), (1+5+8), (1+6+7), (2+3+9), (2+4+8), (2+5+7), (3+4+7), and (3+6+5).

Removing the sets (highlighted in green) that are already covered in Case II leaves me with 11 possible sets.

Total sets = 1+5+11

This gives us 17 possible sets! See the table below.

Case I: 3 occurs 4 times					
8	3	3	3	3	20
Case II: 3 occurs 3 times					
0	3	3	3	11	20
1	3	3	3	10	20
2	3	3	3	9	20
4	3	3	3	7	20
5	3	3	3	6	20
Case III: 3 occurs 2 times					
0	1	3	3	13	20
0	2	3	3	12	20
0	4	3	3	10	20
0	5	3	3	9	20
0	6	3	3	8	20
1	2	3	3	11	20
1	4	3	3	9	20
1	5	3	3	8	20
1	6	3	3	7	20
2	4	3	3	8	20
2	5	3	3	7	20