Let each set have elements $\{a,b,c,d,e\}$, where $a \leq b \leq c \leq d \leq e$.

- Since the mean $= 4$, $\frac{a+b+c+d+e}{5} = 4$

$$\therefore a+b+c+d+e = 20.$$

The sum of the 5 elements must be 20.

- Since the median $= 3$, $c = 3$

We now have $a \leq b \leq 3 \leq d \leq e$.

- Since the mode $= 3$, the number 3 must appear at least ~~twice~~ twice in the set.

Let us consider each of the cases separately.

Case I

- 3 appears exactly twice in each set.

$\therefore b = 3$ or $d = 3$.

$b = 3$:

$\{a,3,3,d,e\}$.

$\therefore a = 1$ or $a = 2$.

$a = 1$:

$\{1,3,3,d,e\}$

$1+3+3+d+e = 20$

$\therefore d+e = 13$

$d > 3$, since 3 appears only twice in this case

$\therefore$ Possible values for d, e:

$\{4,9\}$

$\{5,8\}$

$\{6,7\}$

We can't have $\{7,6\}$, $\{8,5\}$, $\{9,4\}$ ~~$\{4,9\}$~~ since we defined $d \leq e$.

$\boxed{\text{3 possible sets here}}$

$a = 2$:

$\{2,3,3,d,e\}$.

$2+3+3+d+e = 20$

$\therefore d+e = 12$.

the same applies here.

$\therefore$ Possible values for d, e:

$\{4,8\}$

$\{5,7\}$

We can't have $\{6,6\}$ since that would make the set bimodal, whereas we are given the mode $= 3$.

We can't have $\{8,4\}$ or $\{7,5\}$ since $d \leq e$.

$d = 3$:

$\{a,b,3,3,e\}$.

$a, b < 3$.

In fact, $a = 1$ and $b = 2$ because any other combination of numbers would make the set bimodal (e.g. $a=2$, $b=2$). ~~or wo~~

$\{1,2,3,3,e\}$.

$1+2+3+3+e = 20$.

$\therefore e = 11$

$\{1,2,3,3,11\}$.

$\boxed{\text{1 possible set for this sub-case}}$

$\boxed{\text{2 possible sets for this subcase}}$

Case II

- 3 appears exactly 3 times in each set.

-Subcase I

$a=b=c=3$; $\boxed{d, e > 3}$ to ensure 3 appears exactly 3 times in a set

$\{3,3,3,d,e\}$.

$3+3+3+d+e=20$

$\therefore d+e=11$

Possible pairs for d, e are:

$\{4,7\}$

$\{5,6\}$.

2 possible sets for this subcase

We can't have $\{3,8\}$ since that would make 3 appear 4 times in the set which is not what we are considering in this case. We also can't have $\{7,4\}$ or $\{6,5\}$ since $d \leq e$.

-Subcase II

$b=c=d=3$; $\boxed{a<3 \quad , \quad e>3}$ to ensure 3 appears exactly 3 times in a set.

$\{a,3,3,3,e\}$.

$a+3+3+3+e=20$

$\therefore a+e=11$.

The only possible values for a, e are:

~~{2,9}~~ $\{1,10\}$

~~{3~~ $\{2,9\}$.

since $a<3$ and $e>3$.

2 possible sets for this sub-case

- Subcase III.

$c=d=e=3$; $\boxed{a, b < 3}$ to ensure 3 appears exactly 3 times in a set.

$\{a,b,3,3,3\}$.

$a+b+3+3+3=20$

$\therefore a+b=11$

However, $a, b < 3$ and therefore the maximum sum of a and b occurs when $a=b=2$.

$2+2=4$.

The maximum sum is 4 and therefore it is impossible to obtain 11 from $a+b$. $\therefore$ This subcase does not yield any possible sets.

Case III.

· 3 appears exactly 4 times in a set.

- Subcase I.

$a = b = c = d = 3 \quad ; \quad e > 3.$

$\{3, 3, 3, 3, e\}$

$3 + 3 + 3 + 3 + e = 20$

$\therefore e = 8$

$\{3, 3, 3, 3, 8\}.$

$\boxed{\text{1 possible set for this subcase}}$

- Subcase II

$b = c = d = e = 3 \quad ; \quad a < 3$

$\{a, 3, 3, 3, 3\}.$

$a + 3 + 3 + 3 + 3 = 20$

$\therefore a = 8$

However, $a < 3$, and therefore this sub-case does not yield any possible sets.

Case III

· 3 appears 5 times in the set

$\cancel{\therefore a = b = c = d = 3} \quad \cancel{a = b = c = d = 3}$

$\therefore a = b = c = d = e = 3.$

$\{3, 3, 3, 3, 3\}.$

However, the sum of these 5 elements is only 15, whereas the required sum is 20 so that the mean $= 4$.

$\therefore$ This case does not yield any possible sets.

In total, there are $3 + 2 + 1 + 2 + 2 + 1 = \boxed{11}$ sets which satisfy the conditions of the questions:

$\{1, 3, 3, 4, 9\}$ $\{2, 3, 3, 4, 8\}$ $\{3, 3, 3, 4, 7\}$ $\{1, 3, 3, 3, 10\}$ $\{3, 3, 3, 3, 8\}$

$\{1, 3, 3, 5, 8\}$ $\{2, 3, 3, 5, 7\}$ $\{3, 3, 3, 5, 6\}.$ $\{2, 3, 3, 3, 9\}.$

$\{1, 3, 3, 6, 7\}$

$\{1, 2, 3, 3, 11\}.$