# M, M AND M by Eden-Rose

Mean - 4
Mode - 3
Median - 3

If the mean is 4 then the total amount of the 5 numbers must be 20. (5 x 4 = 20.)
If the mode is 3 then there must be more than one 3. The median shows us that our 5 numbers will look a bit like this, with either: ?, ?, 3, 3, ? or ?, 3, 3, ?, ?. Let's start with the first way. It must be 1 and then 2. This is because if the two numbers were 1 and 1 or 2 and 2 then the mode would have to change. We cannot do any higher numbers as we are working in the ascending order. (so it will be easier to calculate the median.)

So we've got up to here: 1,2,3,3...We could do: 1,2,3,3,11. Lets try it the other way now: ?, 3, 3, ?, ?. We could do 1 or 2. Let's be systematic and go with 1. 1,3,3… The next 2 numbers need to add up to 13. We can do 1,3,3,4,9 or 1,3,3,5,8 or 1,3,3,6,7. Now let's try it with 2: 2,3,3,4,8 or 2,3,3,5,7.

What if we try three 3s. We could do: ?,3,3,3,? Or . Let's start with the first one. Let's do 1,3,3,3,10. We could also do 2,3,3,3,9. Now let's try the other way: 3,3,3,?,?. We could do: 3,3,3,5,6 or 3,3,3,4,7 or 3,3,3,3,8 (This uses 4 threes. )

| | | | Position | | |
|---|---|---|---|---|---|
| Combination | 1 | 2 | 3 | 4 | 5 |
| i | 1 | 2 | 3 | 3 | 11 |
| ii | 1 | 3 | 3 | 4 | 9 |
| iii | 1 | 3 | 3 | 5 | 8 |
| iv | 1 | 3 | 3 | 6 | 7 |
| v | 2 | 3 | 3 | 4 | 8 |
| vi | 2 | 3 | 3 | 5 | 7 |
| vii | 1 | 3 | 3 | 3 | 10 |
| viii | 2 | 3 | 3 | 3 | 9 |
| ix | 3 | 3 | 3 | 5 | 6 |
| xi | 3 | 3 | 3 | 4 | 7 |
| xii | 3 | 3 | 3 | 3 | 8 |

| Notes: | |
|---|---|
| | Fixed number as the Median is 3 |
| If the Mean is 4, then the total sum of the 5 numbers will be 4 x 5 = 20 | |
| The Mode is 3, so no other number must repeat more than the number of 3 | |

# M, M and M by Yuna

I worked in a systematic manner in order to ensure that I had all possible solutions to this question. If the mode is 3 then a minimum of two digits must be 3. If the average is 4, then adding all the five numbers should result in 20. Because the median is 3, and there must be two numbers the same or larger than 3 and two the same or smaller than the 3. If the first number was a 1 then you could have 1, 3, 3, 3, 10. Then, working systematically, I added one to the penultimate number and subtracted one from the final number. Using this way, I know that the average will always be 4 and that the mode and median will stay the same as long as I keep at least two 3s. All of my answers so far...

1, 3, 3, 3, 10

1, 3, 3, 4, 9

1, 3, 3, 5, 8

1, 3, 3, 6, 7

Then, continuing with this method, I replaced the 1 at the beginning with a 2. I got these answers…

2, 3, 3, 3, 9

2, 3, 3, 4, 7

2, 3, 3, 5, 7

Once again, I replaced the first digit and made it into a 3. I got the answers…

3, 3, 3, 3, 8

3, 3, 3, 4, 7

3, 3, 3, 5, 6

# M, M and M

After I had found these 10 solutions, I still wasn't convinced that I had found all of the answers. I realised that if at least two 3s were in the sequence, the others would have to equal 14. After thinking about this, I found several options such as 1, 1, 3, 3, 12 which were close but that didn't entirely work. After improving on my incorrect answers and tweaking it slightly each time, I got a group of 5 numbers that did work. They were 1, 2, 3, 3, 11. All of my 11 answers are…

1, 3, 3, 3, 10

1, 3, 3, 4, 9

1, 3, 3, 5, 8

1, 3, 3, 6, 7

2, 3, 3, 3, 9

2, 3, 3, 4, 7

2, 3, 3, 5, 7

3, 3, 3, 3, 8

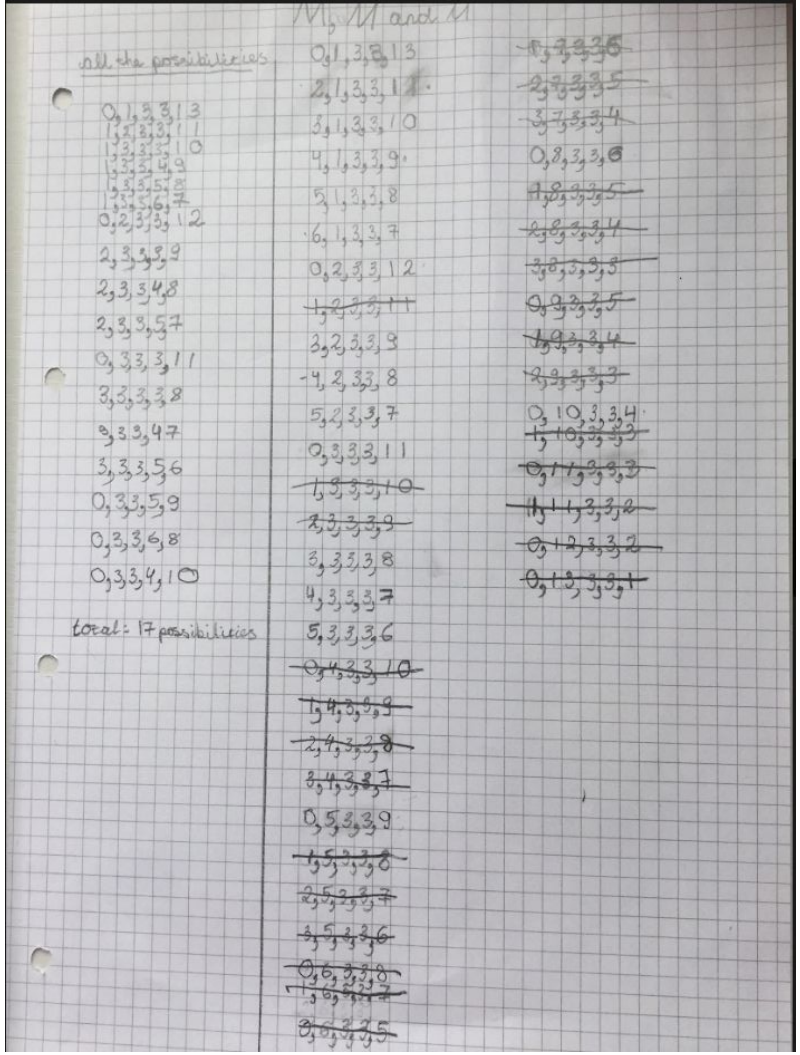
3, 3, 3, 4, 7

3, 3, 3, 5, 6

1, 2, 3, 3, 11

# M,M&M by Vivienne

I worked systematically and these are my only possibilities:
1, 2, 3, 3, 11
1, 3, 3, 3,10
1, 3, 3, 6, 7
1, 3,  3, 5, 8
1, 3, 3, 4, 9
2, 3, 3, 3, 9
2, 3, 3, 4, 8
2, 3, 3, 5, 7
3, 3, 3, 3, 8
3, 3, 3, 4, 7
3, 3, 3, 5, 6
We know that there are 5 numbers and the 3rd has to be three because of the median. So the first and the second one could have either 1, 2 or 3. If we work systematically we can already start with 1 and 2 in the first 2 positions then the 4th would have to be 3 to respect the mode and the fifth one would have to be 11 to respect the mean. Starting with 2 and 2 or 1 and 1 is impossible because the mode would not be respected. We can now try 1, 3 and 3 at the beginning. This leaves us with 2 numbers that have to be equal to 13. This gives us the answers:
1, 3, 3, 6, 7
1, 3,  3, 5, 8
1, 3, 3, 4, 9
Let's move on to 2, 3 and 3. The last 2 numbers have to be equal to 12. This gives us the answers:
2, 3, 3, 3, 9
2, 3, 3, 4, 8
2, 3, 3, 5, 7
Let's move on to 3, 3 and 3. The last 2 numbers have to be equal to 11. This gives us the answers:
3, 3, 3, 3, 8
3, 3, 3, 4, 7
3, 3, 3, 5, 6
These are the only possible answers because otherwise we would not be following the three criteria.




This was my first attempt, but then I realised ‘0’ wasn’t a positive integer so when you rule those out the solution would be 11…
By Elizabeth

# Working Out and Explanation for M, M and M - Jemima

We know that the five numbers need to add up to 20 (4 x 5 = 20), that when you put them in ascending or descending order, the number in the middle is 3 and that the number that appears most frequently is 3.

We can now try to try all the different possibilities. To do this in a systematic way, we can start of with a set of numbers in which 3 appears 4 times and then set of numbers in which 3 appears 3 times and so on. It isn't possible to have five 3s because they wouldn't add up to 20. The possibilities:

Set of numbers in which 3 appears 4 times:

3,3,3,3, ?

We know that the numbers need to add up to 20, so using this we can work out the missing number:

3,3,3,3,?
12 8

The set of numbers is: 3, 3, 3, 3, 8

Set of numbers in which 3 appears 3 times:

3, 3, 3, ?, ?

We know that the numbers must add up to 20 and therefore, as 3 + 3 + 3 = 9, the missing numbers must add up to 11. We can make a list of the numbers that can add up to 11. We also need to remember that the number in the middle when the set of numbers is put in ascending or descending order that the number in the middle is 3, but this doesn't affect the numbers as there will always be a 3 in the middle. Here are the numbers that when added together make 11 (excluding the number 3):

1 + 10, 2 + 9, 4 + 7, 5 + 6.

The set of numbers are: 3,3,3,1,10 **or** 3,3,3,2,9 **or** 3,3,3,4,7 **or** 3,3,3,5,6

The set of numbers in which 3 appears 2 times:

3, 3, ?, ?, ?

When we are working out the solutions to this section we need to be very careful because the missing numbers need to add up to 14 (20 - (3 + 3) = 14), none of the missing numbers can be the same number repeated 2 or more times (otherwise 3 would not be the mode), there need to be 2 numbers lower than three and 1 number higher than than three or 2 numbers higher than three and one number lower than three (if not then 3 wouldn't be the median). Now, we can list the possibilities below starting with the number 1 at the start and then 2 and then 1 and 2 (because as I have explained in the previous sentence, if we had 3 numbers smaller than 3 or 3 numbers smaller then 3 wouldn't the median):

1, 3, 3, 2, 11 1, 3, 3, 4, 9 1, 3, 3, 5, 8 1, 3, 3, 6, 7 2, 3, 3, 4, 8 2, 3, 3, 5, 7

**NOTE:** We can't to a set of numbers starting with 1 and 2 because the set would be 1, 2, 3, 3, 11 which we have already done but in a different order. Due to this problem, there are less solutions as they have already been listed before but in a different order.

**Using these different techniques that I have stated above, we have been able to find all the answers to this problem in a systematic way. I have listed all the sets of numbers possible that satisfy the conditions.**

Mean= 4
4x5=20
Numbers have to sum up to 20

Median =3
2 numbers 3 or smaller
2 numbers 3 or bigger

Mode =3
Has to have 3 more than once=6,9,12,15

1,3,3,3,10
1,3,3,4,9
1,3,3,5,8
1,3,3,6,7
2,3,3,3,9
2,3,3,4,8
2,3,3,5,7
3,3,3,3,8
1,2,3,3,11
3,3,3,4,7
3,3,3,5,6

By Catherine

First of all, if there are 5 numbers and 3 is the median, there must be at least one three in the data set. Since 3 is also the mode, there must be at least two 3s in the set as well. The total sum of the set is 45 = 20. Then, we can draw out what the data set would look like in ascending order.

We can fill in the first two blanks by thinking about what combination of numbers could go there for 3 to be the mode and if the numbers are in ascending order.

We could have:

1. 1 and 1
2. 1 and 2
3. 1 and 3
4. 2 and 2
5. 2 and 3
6. 3 and 3

We can test each outcome to see what possible combinations there are.

1. For 3 to be the single mode, we must have 1, 1, 3, 3, 3 and that does not sum to 20 as desired. Therefore, this is impossible.
2. When 3 is the median, the data set must be 1, 2, 3, 3, 11. This is because 3 needs to be the mode, and the other remaining number that fulfils the total is 11.
3. Here, the beginning of the data set is 1, 3, 3. If we wish to use the 3 again, we can have 1, 3, 3, 3, 10. However, the last two numbers should just sum to 13, starting with 4 add 9. This gives 1, 3, 3, 4, 9 and 1, 3, 3, 5, 8 and 1, 3, 3, 6, 7.
4. For 3 to be the single mode, we must have 2, 2, 3, 3, 3, and this does not sum to 20. It is therefore impossible.
5. The beginning of the data set is 2, 3, 3. If we wish to use the 3 again, we can have 2, 3, 3, 3, 9. However, the last two numbers must sum to 12. The other ways of doing this are 2, 3, 3, 4, 8 and 2, 3, 3, 5, 7. (Note: 2, 3, 3, 6, 6 is not valid because there is no single mode,)
6. The start of the data set looks like 3, 3, 3. If we use the 3 again, we get 3, 3, 3, 3, 8. However, the last two numbers must sum to 11, and the ways of doing this are 3, 3, 3, 4, 7 and 3, 3, 3, 5, 6.

We have found all the solutions as we have seen all the different combinations that can come before the median 3, and used the mean to work out what numbers should come after. The collection of our results are below:

- 1, 2, 3, 3, 11
- 2, 3, 3, 3, 9
- 3, 3, 3, 3, 8

1, 3, 3, 3, 10
2, 3, 3, 4, 8
3, 3, 3, 4, 7

1, 3, 3, 4, 9
2, 3, 3, 5, 7
3, 3, 3, 5, 6

1, 3, 3, 5, 8

1, 3, 3, 6, 7

In total, there are 11 cases.

By Jane