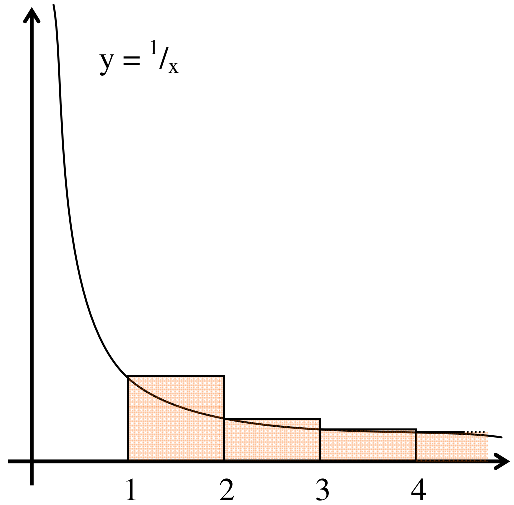
**A)**




$S_n = 1 + {}^1/_2 + {}^1/_3 + {}^1/_4 + \ldots + {}^1/_n > 100$

Each of the areas of the pink rectangles shown above is representative of one fraction in the sum above. This is verified by knowing that each rectangle x has a base of length 1 and a height of length ${}^1/_x$. Every rectangle has an area greater than that of the curve (1/x) it overlaps, as illustrated above (note that this will remain so because the curve 1/x is monotonic for positive numbers), therefore the total area of the rectangles, $S_n$, is greater than the area under the curve:

$$\int_1^n {}^1/_x \, dx = \ln(n) - \ln(1) = \ln(n)$$

Since ln(n) tends to ∞, $S_n$ must also tend to ∞, and thus there is a value of n no larger than $e^{100}$ such that $S_n > 100$.

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**B)** By considering the area under the graph $y = {}^1/_x$, we find that $\int y dx = \ln(x)$. If we evaluate the expression between $x_1 = {}^1/_{n-1}$ and $x_2 = {}^1/_n$, we find the following:

$\ln [x]_{1/n}^{1/(n-1)} = \ln (1/(n-1)) - \ln (1/n)$

$= \ln ((1/(n-1))/(1/n))$

$= \ln (n/(n-1))$

$= \mathbf{\ln (n) - \ln (n-1)} = \ln (1/x_2) - \ln (1/x_1) = \mathbf{\ln (x_1) - \ln (x_2)}$

Thus the area under $y = {}^1/_x$ grows like ln(n), and because, as earlier said, our series grows like $y = {}^1/_x$, the logical conclusion is that the series grows like ln (n).

*Submitted by Ariya & Noah (year 8) of British School of Boston, February 3rd 2009*