

## Cola Can

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Let:  $r$  be the radius of the can

$h$  be the height

$V$  be the volume

$S$  be the surface area

The can is a cylinder so  $V = \pi r^2 h$

$$S = 2\pi r^2 + 2\pi r h = 2\pi r(r+h)$$

Because  $V = 330 \text{ ml} = 330 \text{ cm}^3$  so if  $h$  or  $r$  is given, the other variable is fixed.

$$\textcircled{1} \quad r_1 = 3 \text{ cm}$$

$$h_1 = V/\pi r_1^2 = 330/9\pi = 110/3\pi$$

$$S_1 = 6\pi(3 + 110/3\pi) = 18\pi + 220 \approx 276.52 \text{ cm}^2$$

$$\textcircled{2} \quad h_2 = 10 \text{ cm}$$

$$r_2^2 = 33/\pi$$

$$r_2 = \sqrt{33/\pi}$$

$$S_2 = 2\pi \cdot \sqrt{33/\pi} [\sqrt{33/\pi} + 10] = 66 + \sqrt{13200\pi} \approx 269.59 \text{ cm}^2$$

$$S_1 \approx 276.52 \text{ cm}^2$$

$$S_2 \approx 269.59 \text{ cm}^2$$

$$\therefore S_1 > S_2$$

$\textcircled{3}$  To find the minimum surface area:

In  $S = 2\pi r(r+h)$ , we can replace  $r$  in terms of  $h$

$$h = V/\pi r^2 = 330/\pi r^2$$

We therefore get:

$$S(r) = 2\pi r(r + 330/\pi r^2) = 2\pi r^2 + 660/r$$

We require the surface area to be its minimum value.

Because the function  $S(r)$  is not quadratic, we use trial and error to determine  $r = 3.745$  (3 decimal places),  $h = 7.490$  (3 d.p.) and  $S_{\min} = 264.312$  (3 d.p.).