

For a 4-digit number, $k=3$ \therefore the coefficient on the a term is $\underline{10^3+1}$. Since $k=3$ $\therefore 10^3+1=1001$. 1001 is divisible by 11 and so any 4-digit number is divisible by 11.

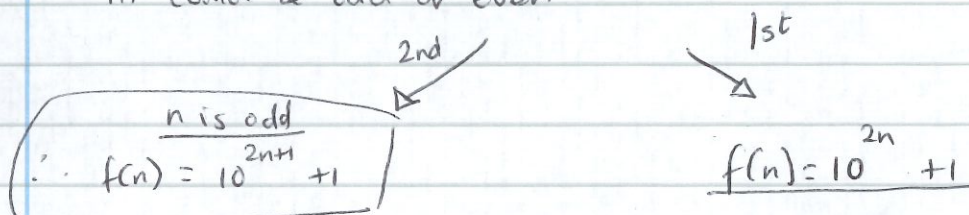
Now let's generalise this for a n -digit number, the coefficient of a is $10^{n-1}+1$. If $10^{n-1}+1$ is a multiple of 11, then ~~the~~ ^{all} n -digit numbers are multiples of 11.

$n-1$ could be odd or even. I noticed that when $n-1$ is odd, then ~~the~~ any n -digit number was a multiple of 11. If $n-1$ is even, then this is not true.

Proof

Let Consider the function $f(n) = 10^m + 1$

m could be odd or even



We will use

Let us try to prove this using induction.

Proposition: $10^{2n}+1$ is always a multiple of 11 for $n \geq 1$

Basis case: $f(n) = 10^{2n+1}$

Let $n=1$ $f(1) = 10^2 + 1$
 $= 101$

101 is not a multiple of 11.

\therefore This ~~for~~ $n=1$ is not true and so 10^{2n+1} is not a multiple of 11 for $n \geq 1$.

2nd $f(n) = 10^{2n+1} + 1$ for m is odd

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Basis case: $f(n) = 10^{2n+1} + 1$ $n=1$ $f(1) = 10^{2+1} + 1$

$$= 10^3 + 1$$

$$= 1001$$

$$= \underline{11 \cdot 91}$$

\therefore The proposition is true for $n=1$

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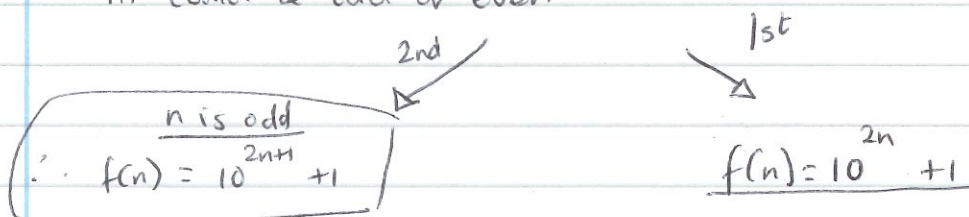
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Assumption

~~Inductive case~~: Assume that the statement is true for $n=k$.

$$\underline{f(k) = 10^{2k+1} + 1}$$

Inductive case: let ~~f(k)~~ $n=k+1$

$$f(k+1) = 10^{2k+1} + 10^{2(k+1)+1} + 1 = \frac{2k+3}{2}$$

For $f(k+1)$ to be a multiple of 11, $f(k+1) - f(k)$ must be a multiple of 11.

$$\begin{aligned} f(k+1) - f(k) &= 10^{2k+3} + 1 - (10^{2k+1} + 1) \\ &= 10^{2k+3} - 10^{2k+1} \\ &= 10^2 \cdot 10^{2k+1} - 10^{2k+1} \\ &= 99 \cdot 10^{2k+1} \\ &= \underline{11 \times 9 \times 10^{2k+1}} \end{aligned}$$

Since $f(k+1) - f(k)$ is a multiple of 11, $f(k+1)$ is a multiple of 11.

Conclusion: if $f(k)$ is a multiple of 11, then $f(k+1)$ is a multiple of 11. Since $f(1)$ is true, the statement $10^{2k+1} + 1$ is always a multiple of 11 for $n \geq 1$ is true.

CONTE EXPLAIN WHAT THIS MEANS

~~The odd~~ For $10^m + 1$, we see that this is only a multiple of 11 when m is odd. ~~odd values of m corresponds~~ m is one less than the number of digits in our original number. So $m+1$ is ~~a~~ always a multiple of 11. Since m is odd, $m+1$ must be even. This leads to the conclusion that all ~~even~~ n -digit numbers, where n is even, obey the rule ~~that~~ of adding the 'back to the queue' version.

- \therefore 3 digits \rightarrow won't work as 3 is odd
- 6 digits \rightarrow will work as 6 is even
- 5 digits \rightarrow won't work as 5 is odd
- 38 digits \rightarrow will work as 38 is even.

Legs Eleven

Jamie Handitye

Firstly, the result does work for all four digit numbers and I will now try to prove this.

Consider a 4 digit number. Let it be m .

$m = 1000a + 100b + 10c + d$ where a, b, c and d are positive integers between 1 and less than 10 and $a \neq 0$.

Let the 'back of the queue' version of m be m'

$$\therefore m' = 1000b + 100c + 10d + a$$

$$m + m' = 1000a + 100b + 10c + d + 1000b + 100c + 10d + a$$

$$= 1001a + 1100b + 110c + 11d$$

$$= 11(91a + 100b + 10c + d)$$

$\therefore m + m'$ will always be a 4-digit number as long as the criteria for a, b, c and d are true.

Now considering an n -digit number.

To do this I looked back at the 4 digit number problem.

For the terms of b, c and d , the terms were in the form

$$10^{k+1} + 10^k \quad \text{e.g. the } b \text{ term was } 1100 = 10^3 + 10^2$$

Look \quad the c term was $110 = 10^2 + 10^1$

Let's see why this always makes ~~the~~ every other term but the a term a multiple of 11.

$$\text{Look at } 10^{k+1} + 10^k = 10^k \cdot 10 + 10^k \\ = \underline{11 \cdot 10^k}$$

This means that every term but the a term is guaranteed to be a multiple of 11.

\therefore For a n -digit number to be a multiple of 11 the ~~last~~ coefficient of the a term must be a multiple of 11. Looking at the 4-digit number case, we can deduce that the a term is always in the form $\underline{10^k + 1}$