For a 4-digit number, $k=3$ $\therefore$ the coefficient on the a term is 10^3+1. ~~Since k~~ $10^3+1=1001$. 1001 is divisible by 11 and so any 4-digit number is divisible by 11.

Now let's generalise this for a n-digit number, the coefficient of a is $10^{\cancel{a}n-1}+1$. If $10^{n-1}+1$ is a multiple of 11, then then all ~~the~~ n-digit numbers are multiples of 11.

$n-1$ could be odd or even. I noticed that when $n-1$ is odd, then ~~the n~~ any n-digit number was a multiple of 11. If $n-1$ is even then this is not true.

Proof

~~Con~~ Consider the function $f(m)=10^m+1$

m could be odd or even

2nd

1st

$$\boxed{\text{n is odd} \quad \therefore f(n)=10^{2n+1}+1}$$

$$f(n)=10^{2n}+1$$

~~We will use~~ Let us try to prove this using induction.

Proposition: $10^{2n}+1$ is always a multiple of 11 for $n\geq 1$

Basis case: $f(n)=10^{2n+1}$

let $n=1$ $\quad f(1)=10^2+1$

$=101$ $\quad$ 101 is not a multiple of 11.

$\therefore$ ~~This fact~~ $n=1$ is not true and so 10^{2n+1} is not a multiple of 11 for $n\geq 1$.

for m is odd

2nd

$$f(n)=10^{2n+1}+1$$

Proposition: $10^{2n+1}+1$ is a multiple of 11 for $n\geq 1$

Basis case: $f(n)=10^{2n+1}$ $\quad n=1$ $\quad f(n)=10^{2+1}+1$

$=10^3+1$

$=1001$

$=11\cdot 91$

$\therefore$ The proposition is true for $n=1$

For a 4-digit number, $k=3$ ∴ the coefficient on the a term is $\cancel{10^3+1}$. ~~Since k i~~ $10^3+1=1001$. 1001 is divisible by 11 and so any 4-digit number is divisible by 11.

Now let's generalise this for a n-digit number, the coefficient of a is $10^{\cancel{n}n-1}+1$. If $10^{n-1}+1$ is a multiple of 11, then then ~~the~~ all n-digit numbers are multiples of 11.

$n-1$ could be odd or even. I noticed that when $n-1$ is odd, then ~~the~~n any n-digit number was a multiple of 11. If $n-1$ is even then this is not true.

Proof

~~Con~~ Consider the function $f(m) = 10^m + 1$

m could be odd or even

2nd

1st

n is odd

$\boxed{\therefore f(n) = 10^{2n+1} + 1}$

$f(n) = 10^{2n} + 1$

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$= 101$ 101 is not a multiple of 11.

∴ ~~This for~~ $n=1$ is not true and so 10^{2n+1} is not a multiple of 11 for $n \geq 1$.

for m is odd

2nd $f(n) = 10^{2n+1} + 1$

Proposition: $10^{2n+1}+1$ is a multiple of 11 for $n \geq 1$

Basis case: $f(n) = 10^{2n+1}$ $n=1$ $f(1) = 10^{2+1}+1$

$= 10^3+1$

$= 1001$

$= \underline{11 \cdot 91}$

∴ The proposition is true for $n=1$

~~Inductive case~~ Assumption: Assume that the statement is true for $n=k$.

$$f(k) = 10^{2k+1}+1$$

Inductive case: Let ~~f(n)~~ ~~f(k+1~~ $n=k+1$

$$f(k+1) = \cancel{10^{2k+1}} \cancel{+1}\ 10^{2(k+1)+1}+1 \quad \cancel{= 2k+3}$$

For $f(k+1)$ to be a multiple of 11, $f(k+1)-f(k)$ must be a multiple of 11.

$$f(k+1)-f(k) = 10^{2k+3}+1-(10^{2k+1}+1)$$
$$= 10^{2k+3}-10^{2k+1}$$
$$= 10^2\, 10^{2k+1} - 10^{2k+1}$$
$$= 99 \cdot 10^{2k+1}$$
$$= 11 \times 9 \times 10^{2k+1}$$

Since $f(k+1)-f(k)$ is a multiple of 11, $f(k+1)$ is a multiple of 11.

Conclusion: if $f(k)$ is a multiple of 11, then $f(k+1)$ is a multiple of 11. Since ~~to~~ $f(1)$ is true, the statement $10^{2k+1}+1$ is ~~always~~ always a multiple of 11 for $n \geq 1$ is true.

~~CONT~~ EXPLAIN WHAT THIS MEANS

~~The odd~~ For 10^m+1, we see that this is only a multiple of 11 when m is odd. ~~Odd values of m corresponds~~ m is one less than the number of digits in our original number. So $m+1$ is ~~a n~~ always a multiple of 11. Since m is odd, $m+1$ must be even. This leads to the conclusion that all ~~even~~ n-digit numbers, where n is even, obey the rule ~~that~~ of adding the 'back to the queue' version.

∴ 3 digits → won't work as 3 is odd
6 digits → will work as 6 is even
5 digits → won't work as 5 is odd
38 digits → ~~a~~ will work as 38 is even.

Legs Eleven

Jamie Handitye

Firstly, the result does work for all four digit numbers and I will now try to prove this.

Consider a 4 digit number. Let it be m.

$m = 1000a + 100b + 10c + d$ where a, b, c and d are ~~positive~~ positive integers ~~between 1 and~~ less than 10 and $a \neq 0$.

Let the 'back of the queue' version of m be m'

$\therefore$ $m' = 1000b + 100c + 10d + a$

$$m + m' = 1000a + 100b + 10c + d + 1000b + 100c + 10d + a$$
$$= 1001a + 1100b + 110c + 11d$$
$$= 11(91a + 100b + 10c + d)$$

$\therefore$ $m + m'$ will always be a 4-digit number as long as the ~~critea~~ criteria for a, b, c and d are true.

Now considering an n-digit number.

To do this I looked back at the 4 digit number problem. For the terms of b, c and d, the terms where in the form $10^{k+1} + 10^{k}$. e.g the b term was $1100 = 10^3 + 10^2$
the c term was $110 = 10^2 + 10^1$

~~Look~~

. Lets see why this always makes ~~the~~ every other term but the a term a multiple of 11.

Look at $10^{k+1} + 10^{k} = 10^{k} \cdot 10 + 10^{k}$
$= \underline{11 \cdot 10^{k}}$

This means that every term but the a term is guranteed to be a multiple of 11.

$\therefore$ For a n-digit number to be a multiple of 11 the ~~art~~ coefficient of the a term must be a multiple of 11. Looking at the 4-digit number case, we can deduce that the a term is always in the form $\underline{10^{k} + 1}$