Name______________________

# Legs 11

Start with any 4 digit number. Create another 4 digit number by taking the 1st digit off and moving it to the last digit (all of the other digits move to the left one space). Prove that if you add these two numbers together, the sum is always divisible by 11.

Example: 1st 4-digit number = 2358
2nd 4-digit number = 3582

The sum is 5940. If you divide by 11 you get the whole number 540.

**Solution:**

Proof by Induction:
Show the 1st case works. Let n = 1000
1000 + 0001 = 1001 Is 1001 divisible by 11? Yes (1001)/(11) = 91.

Now assume that n = k will also work (i.e. is divisible by 11) and show that n = k + 1 is still divisible by 11.

Let k = ABCD where A, B, C, D represent a digit of any 4-digit number. Then…

$k = A(10^3) + B(10^2) + C(10) + D$ The other number would be $B(10^3) + C(10^2) + D(10) + A$

Therefore, the sum is $(A + B)(10^3) + (B + C)(10^2) + (C + D)(10) + (A + D)$

Now, remember that we are assuming this is divisible by 11 and having to show that k + 1 is also divisible by 11.

$k + 1 = A(10^3) + B(10^2) + C(10) + D + 1$
The other number would be $B(10^3) + C(10^2) + (D + 1)(10) + A$

Therefore, the sum is $(A + B)(10^3) + (B + C)(10^2) + (C + D + 1)(10) + (A + D + 1)$

Using distributive property, we get $(A + B)(10^3) + (B + C)(10^2) + (C + D)(10) + 10 + (A + D + 1)$

Thus, it's $(A + B)(10^3) + (B + C)(10^2) + (C + D)(10) + (A + D) + 11$

Since we've already assumed that the yellow highlighted part is divisible by 11, the next case has to also be divisible by 11 since we just ended up adding 11 to the previous case.

This completes the proof that this works for all possible 4 digit numbers.

If left wondering how it proves every case. We proved that the next case (k + 1) has to be divisible by 11 if k is divisible by 11. We also showed that when k = 1000, it is divisible by 11. Therefore, the next case k + 1 = 1001 as the first number would have to work. Since it works, the next would have to work and so on and so on.