If

$$x + \frac{1}{x} = 1$$

Rearrange this into the form

$$x^1 + x^{-1} = 1$$

Let $f(n) = x^n + x^{-n}$. We can investigate the first few terms of $f(n)$.

$$f(1) = x^1 + x^{-1} = 1$$

Starting from $(f(1))^2$,

$$(f(1))^2 = (x^1 + x^{-1})^2 = x^2 + 2x^1x{-}1 + x^{-1} = x^2 + 2 + x^{-2}$$

Since $f(1) = 1$, therefore $(f(1))^2 = 1$. Hence,

$$f(2) = x^2 + x^{-2} = 1 - 2 = -1$$

Further investigations, which can be seen at the end of this solution show that
$f(1) = 1$
$f(2) = -1$
$f(3) = -2$
$f(4) = -1$
$f(5) = 1$
$f(6) = 2$

Also note that $f(0) = 2$ Since the title of the question is 'sextet', I made the assumption that I should be able to work with only six terms. Notice that $f(3)$ and $f(6)$ are both $|2|$. I investigated this as follows.
Proposition 1, $x^3 = -1$.

Take

$$(f(1))^3 = (x^1 + x^{-1})(x^1 + x^{-1})(x^1 + x^{-1}) = 1$$

$$x^3 + 3 + x^{-3} = 1$$

$$x^3 + 2 + x^{-3} = 0$$

Subsituting $y = x^3$ into $x^3 + 2 + x^{-3} = 0$

$$\therefore y + 2 + y^{-1} = 0$$

$$\therefore y^2 + 2y + 1 = 0$$

$$\therefore y = \frac{-2 \pm \sqrt{4 - 4.1.1}}{2.1}$$
$$\therefore y = \frac{-2}{2}$$
$$\therefore y = -1$$
$$\therefore x^3 = -1$$

Proposition 2, $f(m) = 2$ where $m \bmod 6 = 0$.

Let $m = 6k$

$$f(m) = f(6k) = x^{6k} + x^{-6k}$$

Substitute $x = (-1)^{\frac{1}{3}}$ into $f(m)$.

$$f(6k) = (-1)^{\frac{1}{3}^{6k}} + (-1)^{\frac{1}{3}^{-6k}}$$
$$= (-1)^{\frac{6k}{3}} + (-1)^{\frac{-6k}{3}}$$
$$= (-1)^{2^k} + (-1)^{2^{-k}}$$
$$= 1^k + 1^-k$$
$$= 1 + 1 = 2$$

Proposition 3, $f(m) = -2$ where $m \bmod 6 = 3$.

Let $m = 6k - 3$

$$f(m) = f(6k-3) = x^{6k-3} + x^{-6k+3}$$

Substitute $x = (-1)^{\frac{1}{3}}$ into $f(m)$.

$$f(6k-3) = (-1)^{\frac{1}{3}^{6k-3}} + (-1)^{\frac{1}{3}^{-6k+3}}$$
$$= (-1)^{\frac{6k-3}{3}} + (-1)^{\frac{-6k+3}{3}}$$
$$= (-1)^{2k-1} + (-1)^(2k+1)$$
$$= (-1) + (-1) = -2$$

Having see that we can always determine the 3^{rd} and 6^{th} terms in each period, I need to show that we can, based on those two, determine the remaining

terms in the sequence.
To do this I need to prove one of the patterns which can been seen in the series. That is that each term is the sum of the previous term and the following term, ie
For $a > 1$, $f(a) = f(a+1) + f(a-1)$.
To prove this,

$$y = f(a+1) + f(a-1) = (x^{n+1} + x^{-(n+1)}) + (x^{n-1} + x^{-(n-1)})$$

$$y = x^n(x^1 + x^{-1}) + x^{-n}(x^1 + x^{-1})$$

Given $x^1 + x^{-1} = 1$

$$y = x^n + x^{-n}$$

$$\therefore f(a) = f(a+1) + f(a-1)$$

Therefore, $f(a+1) = f(a) - f(a-1)$. This recurrence relationship enables us to generate the entire pattern, which appears to be periodic, with a period of 6.

Proposition 4, for $f(a) = f(a-6)$, where $a > 5$, the sequence is periodic with a period of 6.

For $a = 6$, $f(6) = 2 = f(0)$.
Suppose $f(n) = f(n-6)$, then

$$f(n+6) = x^{n+6} + x^{-(n+6)}$$

$$= x^n x^6 + x{-}nx^{-6}$$
$$= x^n(-1)^{\frac{1}{3}^6} + x^{-n}(-1)^{\frac{1}{3}^{-6}}$$
$$= x^n + x^{-n}$$
$$= f(n)$$

Therefore, the sequence is periodic, period 6.