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Tens

$9^n+1^n=10$, for $n=1$

$9^n+1^n=730$, for $n=3$

I notice that 9^n+1^n is ending in a zero for odd powers of n.

I shall prove this by induction:

Aim $(9^n+1^n)\%10=0\ \forall\ n\%2=1$

1. $9^n+1^n=10$, for $n=1$ Here, "%10" = modulo ten.
   $9^n+1^n=730$, for $n=3$
2. I assume that $(9^k+1^k)\%10=0$, $k\%2=1$
3. $(9^{k+2}+1^{k+2})\%10$

$=(9^2 9^k+1^2 1^k)\%10$

$=[(9^2 9^k)\%10]+[1\%10]$

$=[9^2\%10\times 9^k\%10]+[1\%10]$

$=[1\times 9^k\%10]+[1\%10]$

$=\cancel{9k}[9^k\%10]+[1\%10]$

$=(9^k+1)\%10$

We know that $(9^k+1^k)\%10=10$

$\therefore (9^k+1)\%10=0$

QED

I notice that each of the pairs are number bonds to ten; 7&3, 9&1, 6&4, 2&8. I also notice that for even powers of n, there is a minus sign in between the two terms and for odd powers there is a plus sign.

I will prove these conjectures by using induction.

Here, k is any $\in$ of $\mathbb{Z}^+$, $x\in\{1,2,3,4\}$

Proof 1

$(x^n+(10-x)^n)\%10=0\ \forall n\%2=1$ — RTP

1. I assume that $(x^k+(10-x)^k)\%10=0$, $k\%2=1$
2. $(x^{k+2}+(10-x)^{k+2})\%10$

$=((x^k\times x^2)+((10-x)^k(10-x)^2))\%10$

$=((x^k\times x^2)\%10)+(((10-x)^k(10-x)^2)\%10)$

$=((x^k\%10)\times(x^2\%10))+(((10-x)^k\%10)\times((10-x)^2\%10))$

$((x^k \% 10) \times (x^2 \% 10)) + (((10-x)^k \% 10) \times ((100 \% 10) +$
$(x^2 \% 10) + (20x \% 10)))$
$= ((x^k \% 10) \times (x^2 \% 10)) + (((10-x)^k \% 10) \times (0 +$
$(x^2 \% 10) + 0)$
$= (x^2 \% 10)((x^k \% 10) + ((10-x)^k \% 10))$
$= (x^2 \% 10)((x^k + (10-x)^k) \% 10)$
$= 0$

QED

Proof 2

RTP: $(x^n \cancel{\neq} - \cancel{x}(10-x)^n) \% 10 = 0 \ \forall n \% 2 = 0$

(1) I assume that $(x^k - (10-x)^k) \% 10 = 0, k \% 2 = 0$

(2) $(x^{k+2} - (10-x)^{k+2}) \% 10$
$= ((x^k \times x^2) - ((10-x)^k (10-x)^2))) \% 10$
$= (x^k \times x^2) \% 10 - ((10-x)^k (10-x)^2) \% 10$
$= ((x^k \% 10)(x^2 \% 10)) - (((10-x)^k \% 10)((10-x)^2 \% 10))$
$= ((x^k \% 10)(x^2 \% 10)) - (((10-x)^k \% 10)((100 \% 10) +$
$(x^2 \% 10) + (20x \% 10))$
$= ((x^k \% 10)(x^2 \% 10)) - (((10-x)^k \% 10)(x^2 \% 10))$
$= (x^2 \% 10)((x^k \% 10) - ((10-x)^k \% 10))$
$= (x^2 \% 10)((x^k - (10-x)^k) \% 10))$
$= 0$

QED

In the above proofs, I have ~~written~~ used that
$(ab+cd) \% 10$
$= (ab \% 10) + (cd \% 10)$
$= ((a \% 10)(b \% 10)) + ((c \% 10)(d \% 10))$
However, I should have written
$(ab+cd) \% 10$
$= (ab \% 10) + (cd \% 10)$
$= (((a \% 10)(b \% 10)) \% 10) + (((c \% 10)(d \% 10)) \% 10)$
$= [((a \% 10)(b \% 10)) + ((c \% 10)(d \% 10))] \% 10$
However, I have already proven that the part in box is 0 and therefore it does not matter if the last $\% 10$ is there or not.