

System Speak
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Using the system of equations below:

$$ab=1 \quad (1)$$

$$bc=2 \quad (2)$$

$$cd=3 \quad (3)$$

$$de=4 \quad (4)$$

$$ea=6 \quad (5)$$

I can find b in terms of a using equation (1) by dividing by a to give $b = 1/a$ which I will call equation (6). Therefore b must be a reciprocal of a due to the fact that its product must equal 1. Substituting equation (6) for the value of b into (2) gives $c/a=2$ and so $c=2a$ (7). As a result, $cd=3$ can also be written as $2ad=3$ thus the product of $ad=3/2$. Assuming that a or $d = 1$ and the other equals $3/2$, I can make effective use of trial and error to see whether these values for these two letters prove the rest of the equations true:

Sub $a = 1$ into (1) so $b=1$

Sub $b=1$ into (2) leaving $c=2$

Sub $c=2$ into (3) gives $d=3/2$ (I previously established with assumption that it would make sense if either d or $a = 3/2$ with the other equalling 1)

If $d=3/2$ then $e=4/3/2=8/3$

Sub $e=8/3$ into (5) knowing that $a=1$ doesn't result in an answer of 6 and therefore these set of values are simply incorrect.

Using $a=3/2$ instead of $d=3/2$, $b=1/3/2=2/3$

Sub $b=2/3$ into (2) means that $c=2/2/3$ thus $c=3$.

Knowing already that $d = 1$ if $a = 3/2$, I can check whether this assumption is correct by substituting $c=3$ into (3) and rearranging to find whether $d=1$, which as a matter of fact it does.

If $d=1$ then e must equal 4 and so I can finally check whether these set of values are true by substituting $e=4$ and $a=3/2$ to see if their product is 6: $4*3/2=6$.

Therefore $\{3/2, 2/3, 3, 1, 4\}$ for $\{a, b, c, d, e\}$ satisfy this system. In addition, the system is also satisfied if all the values for $\{a, b, c, d, e\}$ are $\{-3/2, -2/3, -3, -1, -4\}$, in other words all the first set of values if negative also prove the equations true.