NRICH

Mixing Paints

Question 1

1:3 (RED WHITE) Can A

1:7 (RED WHITE) Can B

In a can of x litres, then...

$\frac{2}{8}x$, $\frac{6}{8}x$ (RED, WHITE) CAN A

$\frac{1}{8}x$, $\frac{7}{8}x$ (RED, WHITE) CAN B

Since both have a denominator of 8, let's assume that each can is 8L in total.

2L RED, 6L WHITE — CAN A

1L RED, 7L WHITE — CAN B

Say we want the ratio of 1:4 → $\frac{1}{4}$

Since we want a ratio between TOTAL red volume and TOTAL white volume, we come up with the expression (A and B stand for number of 8L cans of A & B).

$$(2A+B):(6A+7B)$$

Turning this into a fraction

$$\frac{2A+B}{6A+7B}$$

and then into an equation

$$\Rightarrow \frac{2A+B}{6A+7B}=\frac{1}{4}$$

SOLVE!

$$\frac{2A+B}{6A+7B}\times(6A+7B)=\frac{1}{4}\times(6A+7B)$$

$$2A+B=\frac{6A+7B}{4}$$

$$8A+4B=6A+7B$$

$$2A+4B=7B$$

$$\boxed{2A=3B}$$

Now, if we want A & B to be the smallest, then A has to be 3 and B has to be 2

($2A=3B$, $A=\frac{3B}{2}$) In order for a whole #, $A=\frac{3\times 2}{2}$ $(B=2)\Rightarrow A=3$

We can use the same logic for other ratios.

Question 2 is using the same logic, except the expression is now

$$\frac{2C+D}{8C+9D}$$

AS: $\underbrace{\overset{\text{RED}}{2L}, \overset{\text{WHITE}}{8L}}_{\text{CAN C}}$ $\underbrace{\overset{\text{RED}}{1L}, \overset{\text{WHITE}}{9L}}_{\text{CAN D}}$

when 10L cans are used.

Last question: Yes, it is possible to combine two paints of $1:x$ and $1:y$ into $1:z$ (where $x<z<y$).

This is because it is <u>always</u> possible to find a value of the total volume of a can that will produce whole number volumes.

It is also <u>always</u> possible to solve an equation that only involves subtraction, addition and multiplication (as seen in Q1).