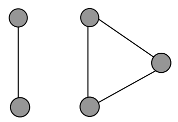
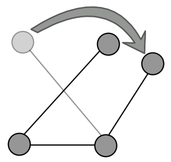
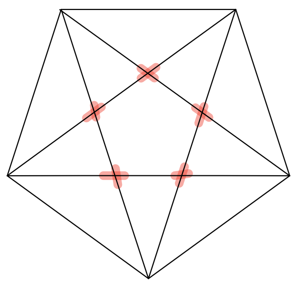
A connected network is a network such that we can get from any vertex to any other by travelling along edges
A planar network is one which can be redrawn where no edges cross.
A cyclic network is one that contains at least one closed loop.
A tree must be connected, planar and acyclic.

This network is disconnected planar and cyclic

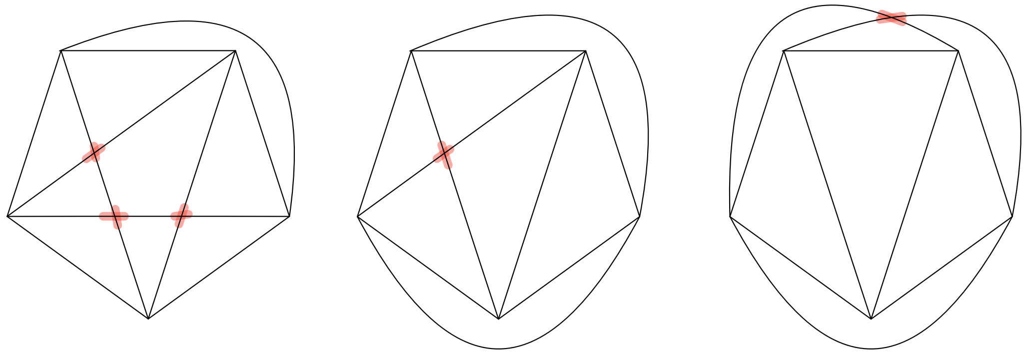


It is impossible to draw a network that is acyclic and non-planar. This is because if there is no closed loop it can always be rearranged so no edges cross.



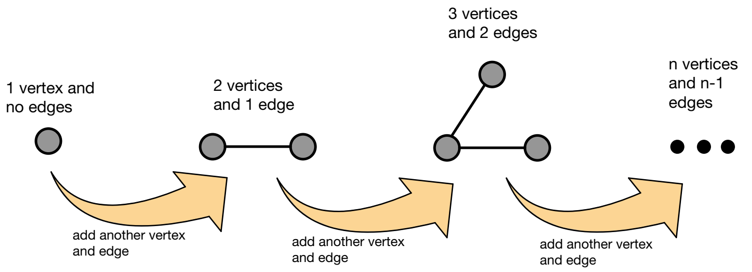


This network is cyclic-it contains a closed loop therefore it is not a tree. It is also non-planar -it cannot be redrawn with no edges crossing



Illustrations that this network is non-planar.

In a tree each edge links two different vertices. There can only be one route between two vertices otherwise a cycle would be created. With one vertex this is clearly impossible. Therefore a tree with 1 vertex has 0 edges. With 2 vertices there is therefore only one edge.



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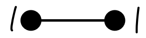
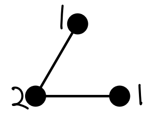
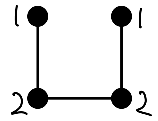
The degrees and sum of degrees of all vertices of all trees with 2-5 vertices



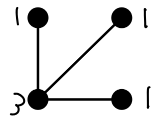
1+1=2



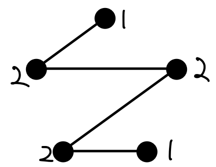
1+2+1=4



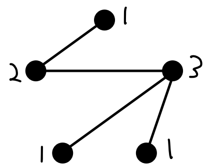
1+2+2+1=6



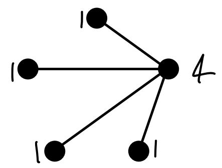
1+3+1+1=6



1+2+2+2=8



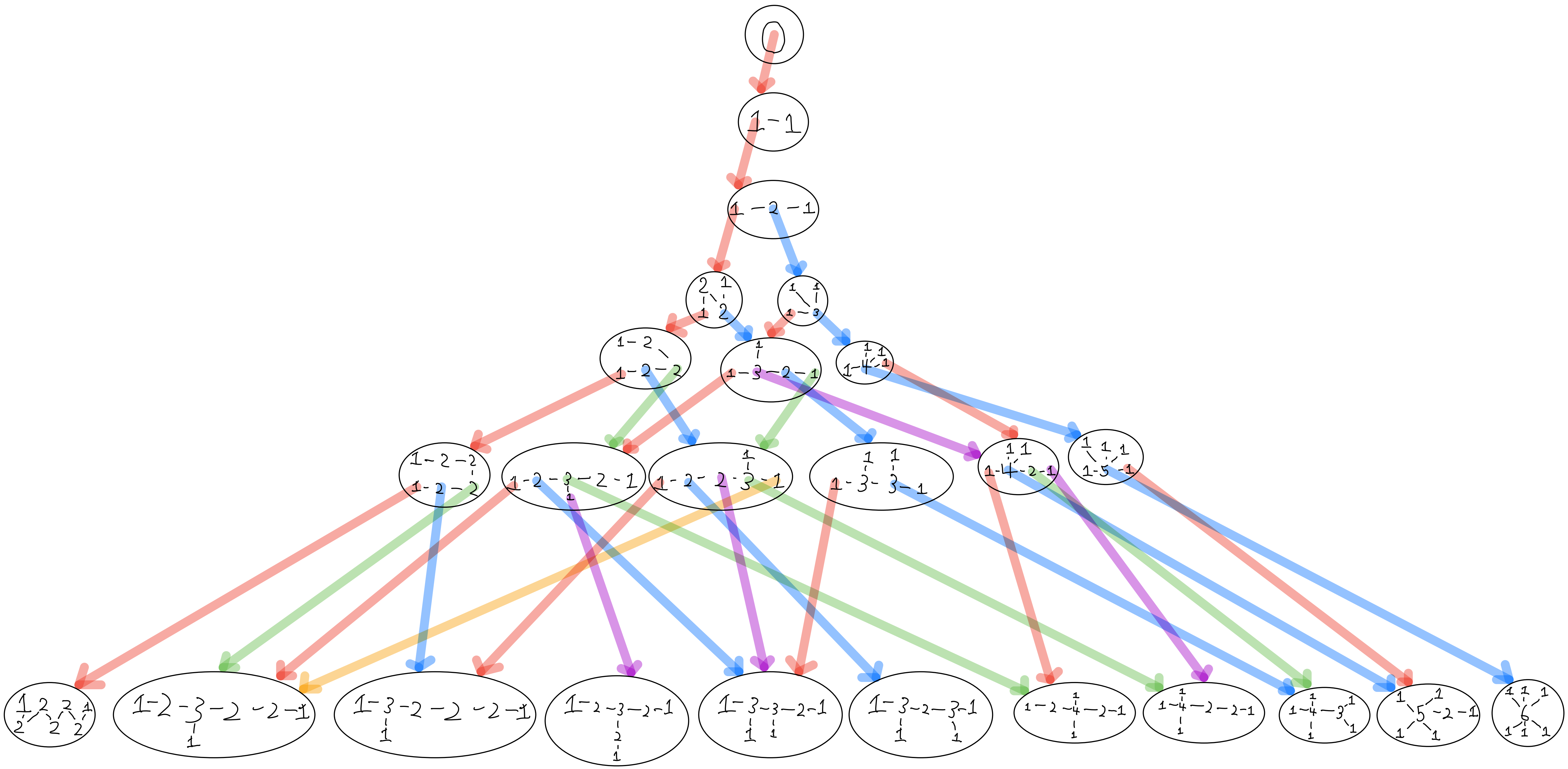
1+2+3+1+1=8



1+4+1+1+1=8

As we can see the sum of degrees is double the number of edges (double the number of vertices minus two). This is because each edge adds one to the degree of both connected vertices.

A directed graph of trees showing where a vertex can be added to one or more previous trees to form each tree



We start with 1 vertex marked with 0 as it is connected to no other nodes. The rest of the vertices are also replaced with their degree. There will be no other vertices marked with 0 as these would be disconnected. All trees with n vertices can be created by adding an extra vertex to one or more trees with n-1 vertices. This new vertex can only be connected to one other vertex, as if it was connected to multiple it would become cyclic. The highlighter shows the ways that a tree can be formed in this way. As all unique possibilities for adding a vertex for each tree are accounted for, there are 11 different trees with 7 vertices.

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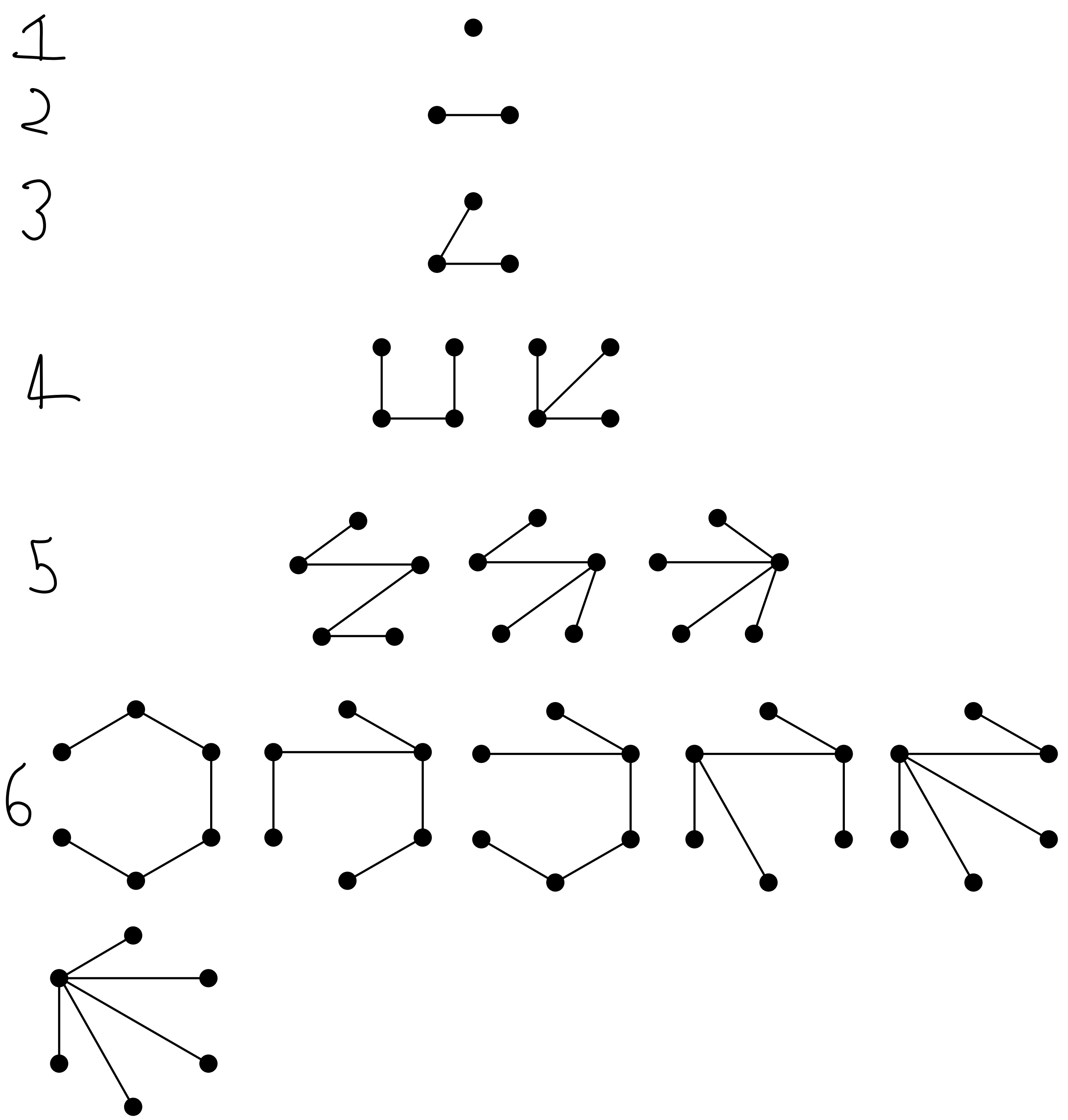
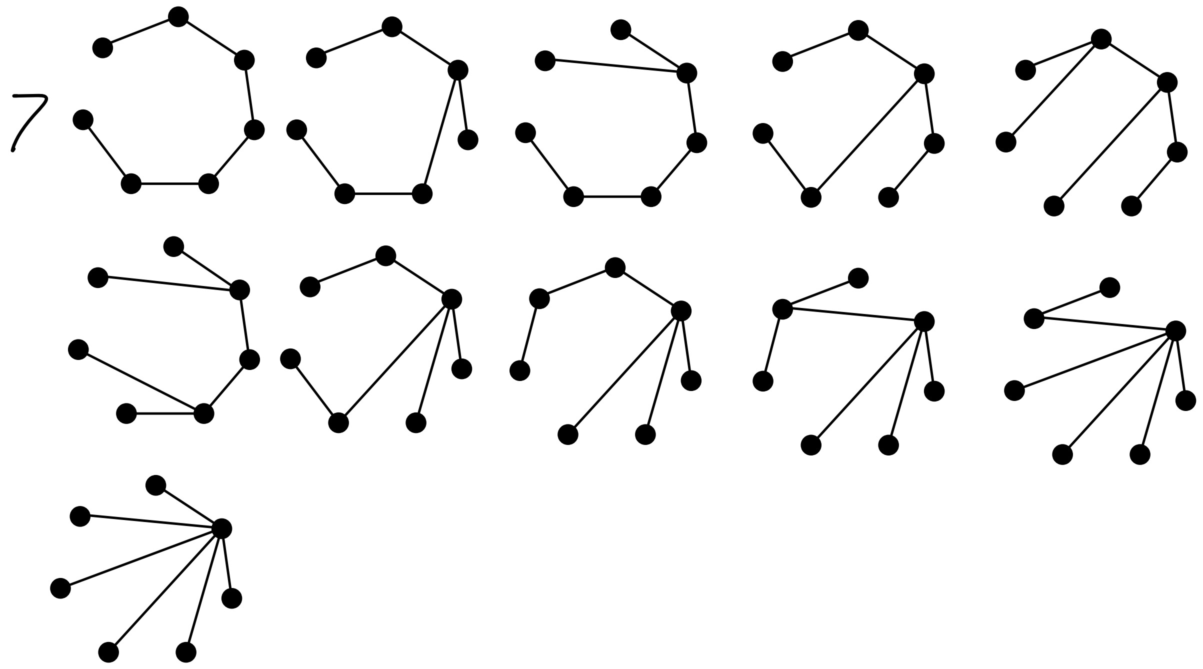
The unique trees with 1 to 7 vertices arranged in regular polygons
in the same order as on the previous page





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