

Rod Fractions

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For any two rods of length 'x' and 'y', when you place y number of rods of length x straight in a line, it will make the same length as placing x number of rods of length y in a line. To make this a bit more clear, let us go through an example.

Ex. You have rods of length 4 and 7. In the problem we do not know the length of the two rods but just to gain familiarity with the problem we assume the lengths for now. So with the two rods of 4 and 7, place 7 such pink (4 unit) rods and 4 of the black (7 units).



We observe that the lengths are equal. And interestingly, this holds true for any two rod length. Using this, we can calculate the ratio of both lengths easily. Intuitively, we can see why the statement in the first line is true because y number of rods of length x makes a total length of xy and vice versa.

$$\underbrace{x + x + x + x + \dots + x}_{y \text{ times}} = y(x) = xy$$

$$\underbrace{y + y + y + \dots + y}_{x \text{ times}} = x(y) = xy$$

For our problem, we can backtrack this statement by placing down many Cuisenaire rods in two rows, each row for one type of rod until the both rows coincide. Now we can count the number rods required for each row and write it in the fraction. And voila!, we get the fraction

Back to the problem. If we are given two rods of unknown lengths, to find the ratio start lining them up in a row and stop when both line up exactly i.e. have the same length exactly at some length . So if you have a Red rod and a Brown rod, start lining them up in 2 different columns until they meet at some length and then count the number of Reds required and Browns required to have them coincide again at a point.



We observe, that they equal in length when there are 4 Red rod and just 1 Brown rod.

$$1 \text{ Brown} = 4 \text{ Reds}$$

$$\therefore \boxed{\text{Red} : \text{Brown} :: 1 : 4}$$

Let's try another set of rods and try if our strategy of lining up works or not. Let the rods be Red and Orange this time.



Once again we observe that 5 Red and 1 Orange are equal in length

$$1 \text{ Orange} = 5 \text{ Reds}$$

$$\therefore \boxed{\text{Red} : \text{Orange} :: 1 : 5}$$

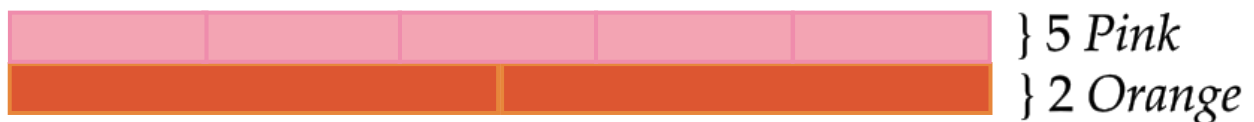
Try another set of rods. Let the rods be Dark green and Blue this time.



Once again we observe that 3 Dark green and 2 Blue are equal in length

$$\therefore \text{Dark green} = \frac{2}{3} \text{Blue}$$

Let another set of rods be Pink and Orange this time.



We observe that 5 Pink and 2 Orange are equal in length

$$\text{Pink} = \frac{2}{5} \text{Orange}$$

In general this rule works for any two Cuisenaire rods until you can align the infinitely many rods and then find the ratio.

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