Different by one

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Let the rods be of difference of length 2. Thus let the lengths be $k, k+2$.

Proof of 'k' not being even:

We first show that if k is an even number the lengths can never differ by 1. Let there be (x) number of k rods and (y) such k+2 length rods.

Then the difference of can be either of the two

$$(x)k - y(k+2) = 1 \qquad \text{OR} \qquad y(k+2) - (x)k = 1$$

We represent this simply by:

$$\Big|(x)k - y(k+2)\Big| = 1$$

If k is an even number, k = 2m.

$$(x)2m - y(2m+2) = 1 \qquad \text{OR} \qquad y(2m+2) - (x)2m = 1$$

$$2mx - 2y(m+1) = 1 \qquad \text{OR} \qquad 2y(m+1) - 2mx = 1$$

$$2(mx - y(m+1)) = 1 \qquad \text{OR} \qquad 2(y(m+1) - mx) = 1$$

$$2\underbrace{(mx - y(m+1))}_{\text{Always an Integer}} = 1 \qquad \text{OR} \qquad 2\underbrace{(y(m+1) - mx)}_{\text{Always an Integer}} = 1$$

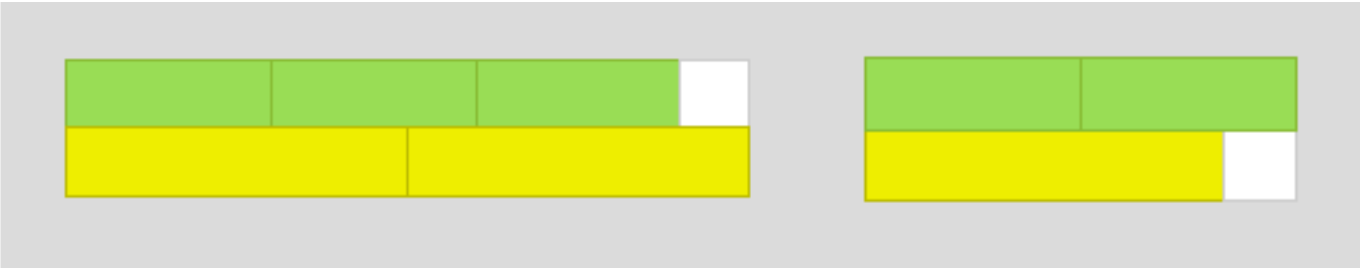
Since the quantities mentioned are Always an integer, and never a fraction, we can never get 1 by their differences

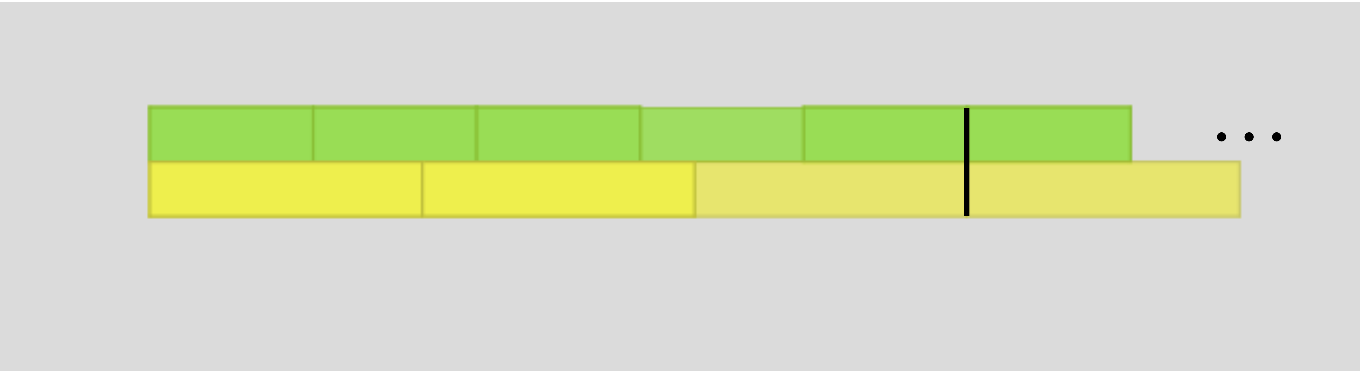
Proof of ‘k’ being odd:

If k is not even, then k must be odd.
Then k+2 will also be odd.

Numerical example:

To have a better idea of the result, let us check an example from the problem and observe a few things.





Firstly we observe that after 15 units the lengths align, and after that the arrangement is same as the beginning until the length reaches 30 and it repeats in a cycle. This happened because we have used five 3 unit rods and three 5 unit rods. From the Rod fraction activity we can see why they align.

In general this happens because we have k+2 such k unit rods and k such k+2 unit rods. This corresponds to the x and the y.

$$(k+2)(k) = (k)(k+2)$$

Let us take another example. If the rods are of length 13 and 15.

The main difference is 2 at first

When we use two rods of 13 and two rods of 15 and arrange them in a line, we get the difference to be 4 and we continue

$$2(15) - 2(13) = 2(2)$$

$$3(15) - 3(13) = 3(2)$$

$$4(15) - 4(13) = 4(2)$$

$$5(15) - 5(13) = 5(2)$$

$$6(15) - 6(13) = 6(2)$$

$$7(15) - 7(13) = 7(2)$$

When we get the difference to be 14, we can do either two things

Case 1:

Add the value of 13

$$7(15) - \left(7(13) + 13\right)$$ Total eight 13 length rods

$$7(15) - 8(13)$$

$$7(15) - 8(13) = 1$$

Case 2:

Subtract the value 15

$$\left(7(15) - 15\right) - 7(13)$$ Total six 15 length rods

$$6(15) - 7(13)$$

$$6(15) - 7(13) = -1$$

Since the difference is positive, -1 is just the order of subtraction.

Generalise to k:

If k and $k+2$ are the two odd length rods with difference 2, we represent them with:

$$k = 2m - 1$$

$$k + 2 = 2m + 1$$

To achieve the similar cases and the result we have derived, we can get the average of k and $k+2$ and divide it by the difference which is 2 units

$$\text{Average} = \frac{(2m-1) + (2m+1)}{2}$$

$$\text{Average} = 2m$$

$$\frac{\text{Average}}{\text{Difference}} = \frac{2m}{2} = m$$

Use (m) rods of k and k+2 length.

<u>Case 1:</u>

Add one k unit rod to the line. We get the difference

$$\text{Difference} = \left|\left((m)(k+2) - (m+1)(k)\right)\right|$$

(Replace k and k+2 with it's defined values)

$$= \left|\left((m)(2m+1) - (m+1)(2m-1)\right)\right|$$

$$= \left|\left((2m^2 + m) - (2m^2 + m - 1)\right)\right|$$

$$= \left|\left(2m^2 + m - 2m^2 - m + 1\right)\right|$$

$$= \left|\left(\cancel{2m^2 + m - 2m^2 - m} + 1\right)\right|$$

$$= 1$$

Case 2:

Subtract $k + 2$ from the line of rods. We again get the difference

$$\begin{aligned}
\text{Difference} &= \left|\left((m-1)(k+2) - (m)(k)\right)\right| \\
&= \left|\left((m-1)(2m+1) - (m)(2m-1)\right)\right| \\
&= \left|\left((2m^2 - m - 1 - (2m^2 - m)\right)\right| \\
&= \left|\left(\cancel{2m^2 - m} - 1 \cancel{-2m^2 + m}\right)\right| \\
&= \left|-1\right| \\
&= 1
\end{aligned}$$