

Lines are graphed onto a graph to make questions simpler
 Using point A as coordinates (0,0)

focusing on the squares:

a = length of line AM, b = length of line MB

finding where lines AD and extended line BE intersect

Gradient of line AD, $\frac{b}{a}$

Equation of line AD, $y = \frac{b}{a}x$

Gradient of line BE, $-\frac{a}{b}$

y intercept, fitting to equation $0 = -\frac{a}{b}(a+b) + c$ (when y=0)

$$c = \text{y-intercept} = \frac{a(a+b)}{b}$$

Equation of line BE, $y = -\frac{a}{b}x + \frac{a(a+b)}{b}$

interception, using substitution method

$$\frac{b}{a}x = -\frac{a}{b}x + \frac{a(a+b)}{b}$$

– multiplying (ab)

$$b^2x = -a^2x + a^2(a+b)$$

– move the x to one side

$$b^2x + a^2x = a^2(a+b)$$

– factorize

$$(b^2 + a^2)x = a^2(a+b)$$

– rearrange to make “x” the subject

$$x = \frac{a^2(a+b)}{a^2 + b^2}$$

find y,

$$y = \frac{b}{a}x$$

– sub x

$$y = \frac{b}{a} \times \frac{a^2(a+b)}{a^2 + b^2}$$

– solve

$$y = \frac{ab(a+b)}{a^2 + b^2}$$

$$x = \frac{a^2(a+b)}{a^2 + b^2}$$

$$y = \frac{ab(a+b)}{a^2 + b^2}$$

Focusing at the circles:

Let’s call the Center of circle FAMEP, G, and circle DCBMP, H.

the diameter and radius of circle FAMEP can be found by the diagonal of square FEMA

$$\text{Diameter} = \sqrt{2}a \qquad \text{Radius} = \frac{a}{\sqrt{2}}$$

this tells us that point P is $\frac{a}{\sqrt{2}}$ away from point G

The middle point of circle FAMEP is $\left(\frac{a}{2}, \frac{a}{2}\right)$

Similarly, the diameter and radius of circle DCBMP can be found by the diagonal of square DCBM

$$\text{Diameter} = \sqrt{2}b \qquad \text{Radius} = \frac{b}{\sqrt{2}}$$

this tells us the point P is $\frac{b}{\sqrt{2}}$ away from point H

The middle point of circle DCBMP is $\left(\frac{2a+b}{2}, \frac{b}{2}\right)$, this is found by the middle point of the square.

calculating the shortest distance between the two circles:

$$\frac{2a+b}{2}, \frac{a}{2}, \frac{b}{2}$$

$$\frac{a+b}{2}, \frac{b-a}{2}$$

$$\left(\frac{a+b}{2}\right)^2 + \left(\frac{b-a}{2}\right)^2 = \text{distance}^2$$

$$\frac{a^2 + 2ab + b^2 + a^2 - 2ab + b^2}{4} = \text{distance}^2$$

$$\frac{2a^2 + 2b^2}{4} = \text{distance}^2$$

$$\text{distance} = \sqrt{\frac{2a^2 + 2b^2}{4}}$$

$$\text{distance} = \frac{\sqrt{a^2 + b^2}}{\sqrt{2}}$$

From the circles, we can make a triangle GHP
make a perpendicular line from point P to line GH. Call the point of intersection J

from this, we can form some formulas

$$GJ^2 + PJ^2 = GP^2$$

$$HJ^2 + PJ^2 = HP^2$$

$$GH = GJ + HJ$$

from this, we can solve GJ

$$GJ^2 = GP^2 - PJ^2$$

$$GJ^2 = GP^2 - (PH^2 - HJ^2)$$

$$GJ^2 = GP^2 - PH^2 + HJ^2$$

$$GJ^2 = GP^2 - PH^2 + (GH - GJ)^2$$

$$GJ^2 = GP^2 - PH^2 + GH^2 - 2GH \times GJ + GJ^2$$

$$GJ^2 + 2GH \times GJ - GJ^2 = GP^2 - PH^2 + GH^2$$

$$2GH \times GJ = GP^2 - PH^2 + GH^2$$

$$GJ = \frac{GP^2 - PH^2 + GH^2}{2GH}$$

now sub the the equations in from the top

$$GJ = \frac{GP^2 - PH^2 + GH^2}{2GH}$$

$$GJ = \frac{\left(\frac{a}{\sqrt{2}}\right)^2 - \left(\frac{b}{\sqrt{2}}\right)^2 + \sqrt{\frac{a^2 + b^2}{2}}}{\sqrt{4} \times \sqrt{\frac{a^2 + b^2}{2}}}$$

$$GJ = \frac{\frac{a^2}{2} - \frac{b^2}{2} + \frac{a^2 + b^2}{2}}{\sqrt{2} \times \sqrt{a^2 + b^2}}$$

$$GJ = \frac{a^2 - b^2 + a^2 + b^2}{2 \times \sqrt{2} \times \sqrt{a^2 + b^2}}$$

$$GJ = \frac{2a^2}{2 \times \sqrt{2} \times \sqrt{a^2 + b^2}}$$

$$GJ = \frac{a^2}{\sqrt{2} \times \sqrt{a^2 + b^2}}$$

After working out what GJ, we could work out the fraction of the distance of the straight line between points G and H

$$GJ = \frac{a^2}{\sqrt{2} \times \sqrt{a^2 + b^2}}$$

$$\frac{GJ}{GH} = \frac{\frac{a^2}{\sqrt{2} \times \sqrt{a^2 + b^2}}}{\frac{\sqrt{a^2 + b^2}}{\sqrt{2}}}$$

$$\frac{GJ}{GH} = \frac{a^2 \times \sqrt{2}}{\sqrt{a^2 + b^2} \times \sqrt{a^2 + b^2} \times \sqrt{2}}$$

$$\frac{GJ}{GH} = \frac{a^2}{a^2 + b^2}$$

we can use this to find the point J on the straight line GH

Starting Point (Co-ordinates) + Length from Starting point to Finish point X fraction of the

line ($\frac{GJ}{GH}$)

Point G (x,y) + change of x and change of y X $\frac{GJ}{GH}$

From this we get two equations, one for x and one for y

Point G (x co-ordinate) + change of x from G to H, X $\frac{GJ}{GH}$

$$x = \frac{a}{2} + \frac{a+b}{2} \times \frac{a^2}{a^2 + b^2}$$

$$x = \frac{a}{2} + \frac{a^2(a+b)}{2(a^2 + b^2)}$$

$$x = \frac{a(a^2 + b^2)}{2(a^2 + b^2)} + \frac{a^2(a+b)}{2(a^2 + b^2)}$$

$$x = \frac{a^3 + ab^2 + a^3 + a^2b}{2(a^2 + b^2)}$$

$$x = \frac{2a^3 + ab^2 + a^2b}{2(a^2 + b^2)}$$

Point G (y co-ordinate) + change of y from G to H X $\frac{GJ}{GH}$

$$y = \frac{a}{2} + \frac{b-a}{2} \times \frac{a^2}{a^2+b^2}$$

$$y = \frac{a}{2} + \frac{a^2(b-a)}{2(a^2+b^2)}$$

$$y = \frac{a(a^2+b^2)}{2(a^2+b^2)} + \frac{a^2(b-a)}{2(a^2+b^2)}$$

$$y = \frac{a^3+ab^2-a^3+a^2b}{2(a^2+b^2)}$$

$$y = \frac{ab^2+a^2b}{2(a^2+b^2)}$$

$$y = \frac{ab(a+b)}{2(a^2+b^2)}$$

Point J has the co-ordinates of $\left(\frac{2a^3+ab^2+a^2b}{2(a^2+b^2)}, \frac{ab(a+b)}{2(a^2+b^2)}\right)$

Find point P from Point M and J

the line GH is perpendicular bisector to the line PM, splitting the line in half, the fraction of

$\frac{2}{1}$

this point from the line, is 1 or 2

the line MJP is a straight line,

this changes the formula slightly

Starting point (point M) + change in x and y X 2
for X, it is

$$x = a + 2\left(\frac{2a^3+ab^2+a^2b}{2(a^2+b^2)} - a\right)$$

$$x = a + \frac{2a^3+ab^2+a^2b}{a^2+b^2} - 2a$$

$$x = \frac{2a^3+ab^2+a^2b}{a^2+b^2} - a$$

$$x = \frac{2a^3+ab^2+a^2b}{a^2+b^2} - \frac{a(a^2+b^2)}{a^2+b^2}$$

$$x = \frac{2a^3+ab^2+a^2b-a(a^2+b^2)}{a^2+b^2}$$

$$x = \frac{2a^3+ab^2+a^2b-a^3-ab^2}{a^2+b^2}$$

$$x = \frac{a^3+a^2b}{a^2+b^2}$$

$$x = \frac{a^2(a+b)}{a^2+b^2}$$

For the Y, it is

$$y = 0 + 2\left(\frac{ab(a+b)}{2(a^2+b^2)} - 0\right)$$

$$y = \frac{ab(a+b)}{a^2+b^2}$$

From Focusing on the circles, we could say find Point P's co-ordinates

$$x = \frac{a^2(a+b)}{a^2+b^2}$$

$$y = \frac{ab(a+b)}{a^2+b^2}$$

From these two different focuses on finding their point of intersection, they both intersection takes hold in the same place. This would mean that they would all meet at point P.