

Gist of Gems

Ark Academy Solved by Bipanshu Sharma, Alicia Lawrence and Olivia Lee

- Let r be the value of one ruby
- Let d be the value of one diamond
- Let s be the value of one sapphire
- Let p be the value of one pearl.

Then:

- Jeweller 1 has a collection of value $8r$.
- Jeweller 2 has a collection of value $10s$.
- Jeweller 3 has a collection of value $100p$.
- Jeweller 4 has a collection of value $5d$.

So, as they give one jewel of their own to each of the other jewellers:

$$\begin{aligned}\text{Jeweller 1} &= 8r - 3r + d + s + p \\ &= 5r + d + s + p\end{aligned}$$

$$\begin{aligned}\text{Jeweller 2} &= 10s - 3s + d + r + p \\ &= 7s + d + r + p\end{aligned}$$

$$\begin{aligned}\text{Jeweller 3} &= 100p - 3p + s + r + d \\ &= 97p + s + r + d\end{aligned}$$

$$\begin{aligned}\text{Jeweller 4} &= 5d - 3d + s + r + p \\ &= 2d + s + r + p\end{aligned}$$

To find the relative value we have to find out what each gem costs.

Due to all 4 jewellers having an equal amount:

$$\begin{array}{c} \textcircled{1} \qquad \qquad \qquad \textcircled{3} \\ 5r + d + s + p = 97p + s + r + d \\ \begin{array}{cccc} -r & -d & -s & -p \end{array} \quad \begin{array}{cccc} \begin{array}{c} p \\ -p \end{array} & \begin{array}{c} -s \\ -s \end{array} & \begin{array}{c} -r \\ -r \end{array} & \begin{array}{c} -d \\ -d \end{array} \end{array}$$

$$4r = 96p \\ \div 4 \qquad \div 4$$

$$r = 24p$$

$$\begin{array}{c} \textcircled{1} \qquad \qquad \qquad \textcircled{2} \\ 5r + d + s + p = 7s + d + r + p \\ \begin{array}{cccc} -r & -d & -s & -p \end{array} \quad \begin{array}{cccc} \begin{array}{c} s \\ -s \end{array} & \begin{array}{c} -d \\ -d \end{array} & \begin{array}{c} -r \\ -r \end{array} & \begin{array}{c} -p \\ -p \end{array} \end{array}$$

$$4r = 6s$$

because $4r = 96p$

$$96p = 6s \\ \div 6 \qquad \div 6$$

$$16p = s$$

$$\begin{array}{c} \textcircled{2} \qquad \qquad \qquad \textcircled{4} \\ 7s + d + r + p = 2d + s + r + p \\ \begin{array}{cccc} -s & -d & -r & -p \end{array} \quad \begin{array}{cccc} \begin{array}{c} d \\ -d \end{array} & \begin{array}{c} -s \\ -s \end{array} & \begin{array}{c} -r \\ -r \end{array} & \begin{array}{c} -p \\ -p \end{array} \end{array}$$

$$6s = d$$

as $6s = 96p$,

$$96p = d$$

This means that the relative value is:

$$d = 96p = 6s = 4r$$

As we know what the relative value is, we have chosen to convert each gem's value to pearls, so that we avoid using fractions and decimals.

It means that:

• Jeweller 1 has 8 rubies ($8r$)

$$4r = 96p$$

$$8r = 96p \times 2$$

$$\underline{8r = 192p}$$

• Jeweller 2 has 10 sapphires ($10s$)

As we know $6s = 96p$ and $s = 16p$
 $10s$ must be equal to $160p$

$$\underline{10s = 160p}$$

• Jeweller 3 has 100 pearls ($100p$)

• Jeweller 4 has 5 diamonds ($5d$)

$$96 \times 5 = 480$$

As we know, after the exchange of gems, each jeweller has the same value.

As Jeweller 1 ends up with $5r + d + s + p$, we can convert this into pearls which is $120p + 96p + 16p + p = 233p$

Jewellers 1, 2, 3 and 4 ended it in equal value of 233 pearls. To find the value of what each jeweller lost or gained, we have to find out how to get from the original value to the new value of 233 pearls.

This means that:

- Jeweller 1 has gained 41 pearls, we see this as $233 - 192 = 41$
- Jeweller 2 has gained 73 pearls, we see this as $233 - 160 = 73$
- Jeweller 3 has gained 133 pearls, we see this as $233 - 100 = 133$
- Jeweller 4 has lost 247 pearls, we see this as $480 - 233 = 247$

Each jeweller has gained value apart from Jeweller 4, who has lost value.