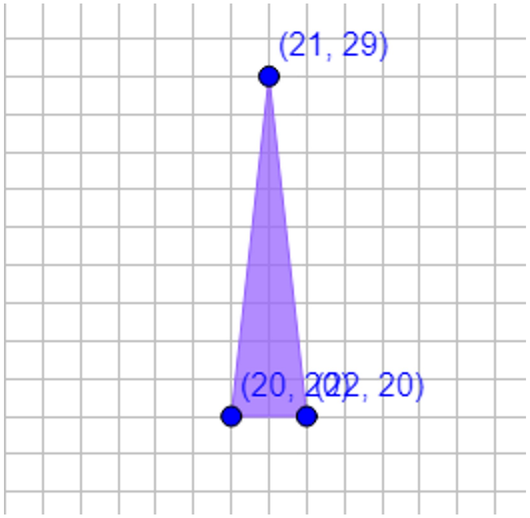
Isosceles Triangles

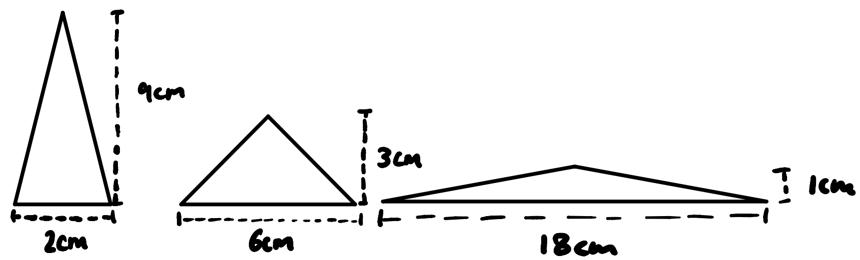
Friday, January 27, 2023 7:58 PM



There can be infinitely as many isosceles triangles with the area as 9 cm^2. However, we know that:

- Any of the three vertices of the triangle must be (20,20)
- The points of the triangle can only have coordinates where x and y are integers

Since isosceles triangles have two equal sides, it must have a base of a length divisible by 2 in order to have integer coordinates!



$\frac{9\times 2}{2} = 9cm^2$ $\frac{6\times 3}{2} = 9cm^2$ $\frac{18\times 1}{2} = 9cm^2$

Base cannot be lengths of 4, 8 or above 18 as:

$\frac{4\alpha}{2} = 9$

$2\alpha = 9$

$\alpha = 4.5$

height isn't an integer

$\frac{8\beta}{2} = 9$

$4\beta = 9$

$\beta = 2.25$

height isn't an integer

$20 > 18$

$\frac{20d}{2} = 9$

$10d = 9$

$d = 0.9$

$0.9 < 1$

Base of greater than 18 will have no integer height

$\frac{12e}{2} = 9$

$6e = 9$

$e = \frac{3}{2}$

not integer

$\frac{14f}{2} = 9$

$7f = 9$

$f = \frac{9}{7}$

not integer

$\frac{10x}{2} = 9$

$5x = 9$

$x = \frac{9}{5}$

not integer

$\frac{16g}{2} = 9$

$8g = 9$

$g = \frac{9}{8}$

$\frac{18h}{2} = 9$

$9h = 9$

$h = 1$

$\underbrace{g = \frac{7}{8}}_{\text{isn't integer}}$ $\underbrace{h = 1}_{\text{largest value for base is 18.}}$ $\underbrace{}_{\text{not integer}}$

Since the largest base has length 18, there are only 3 suitable triangles that meet all the requirements.

There are three points of a Δ and any one of them must be on point (20,20), then:

$$\underbrace{3}_{\text{amt. of } \Delta s} \times \underbrace{3}_{\text{points}} = 9$$

Triangle can go in all 4 directions, so there are:

$$\underbrace{9}_{\text{amt. of } \Delta s} \times \underbrace{4}_{\text{directions}} = 36$$

There are 36 isosceles triangles that can be made with the requirements.