

Overturning Fracsum

Yuk-Chiu

TO..... FROM..... DATE..... NO.....

Question

Three equations:

① $\frac{xy}{x+y} = \frac{1}{2}$

② $\frac{yz}{y+z} = \frac{1}{3}$

③ $\frac{xz}{x+z} = \frac{1}{7}$

Solution

Rearrange equation ①.

$$\frac{xy}{x+y} = \frac{1}{2}$$

$$2xy = x+y$$

$$2 = \frac{x+y}{xy}$$

$$2 = \frac{x}{xy} + \frac{y}{xy}$$

$$2 = \frac{1}{y} + \frac{1}{x}$$

Rearrange equation ②.

$$\frac{yz}{y+z} = \frac{1}{3}$$

$$3yz = y+z$$

$$3 = \frac{y+z}{yz}$$

$$3 = \frac{1}{y} + \frac{1}{z}$$

Rearrange equation ③.

$$\frac{xz}{x+z} = \frac{1}{7}$$

$$7xz = x+z$$

$$7 = \frac{x+z}{xz}$$

$$7 = \frac{1}{x} + \frac{1}{z}$$

③ - ② - ①

$$7-3-2 = \frac{1}{x} + \frac{1}{z} - \frac{1}{y} - \frac{1}{z} - \frac{1}{y} - \frac{1}{x}$$

$$2 = -\frac{2}{y}$$

$$-1 = y$$

$$y = \underline{\underline{-1}}$$

Use equation ① to find x.

$$2 = \frac{1}{-1} + \frac{1}{x}$$

$$3 = \frac{1}{x}$$

$$x = \underline{\underline{\frac{1}{3}}}$$

Use equation ② to find z.

$$3 = \frac{1}{-1} + \frac{1}{z}$$

$$4 = \frac{1}{z}$$

$$z = \underline{\underline{\frac{1}{4}}}$$

$$\therefore x = \frac{1}{3}, y = -1, z = \frac{1}{4}$$