20/April/2024.

10mm Squares

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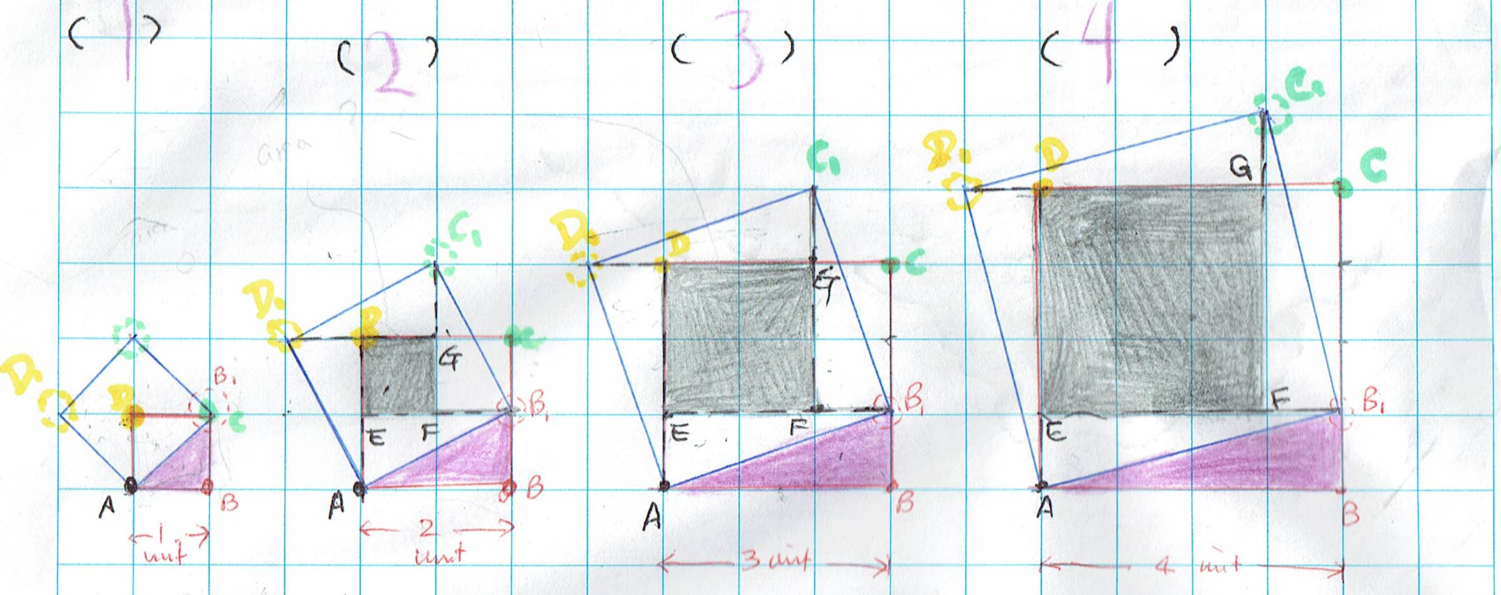
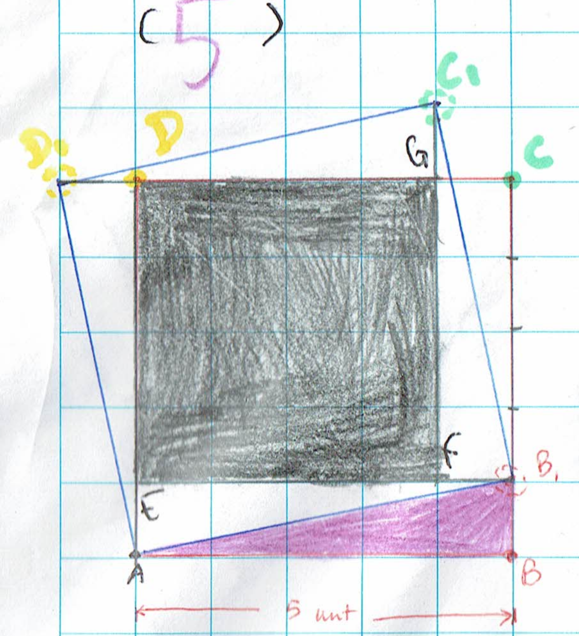
# Tilted Squares

Tilted 1 unit



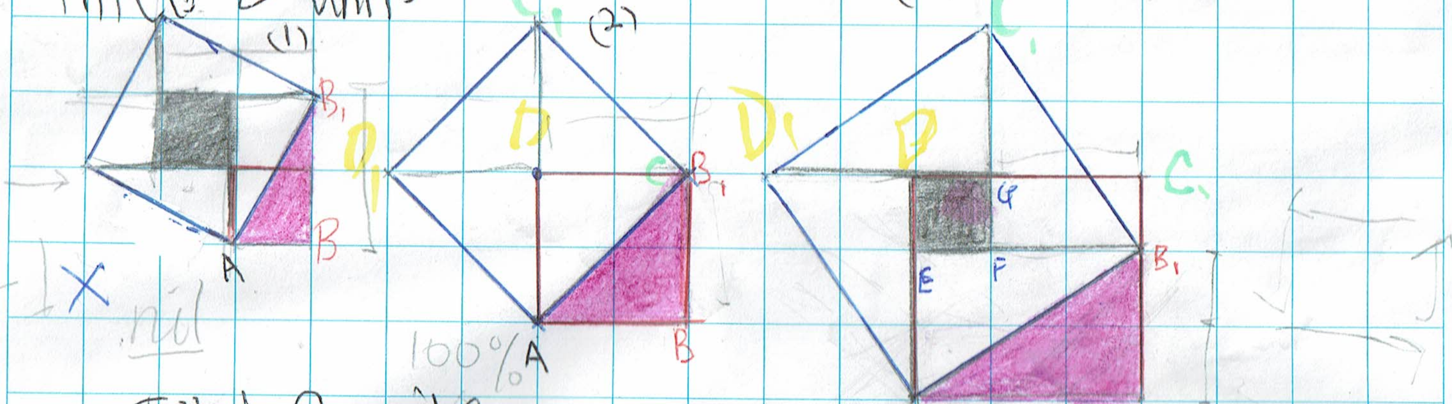




| Item No. | Area of original square ABCD (unit²) | 1△ABB₁ = ? sq ABCD | Area of tilted square AB₁C₁D₁ (unit²) | | | Note: Triangle ABB₁, Triangle AB₁E, FB₁C₁, GC₁D₁, AD₁D, are congruent |
|---|---|---|---|---|---|---|
| 1 | 1×1 It has ~~two~~ 2 triangles, ABB₁ | $\frac{1}{2}$ □ABCD | 0×0+4(△ABB₁) | $= 0 + 4(\frac{1}{2}$ □ABCD) $= 4(\frac{1}{2} \times 1 \times 1)$ $= 2$ | 1×1+1×1 | Original Square Area ABCD + 1 unit square |
| 2 | 2×2 It has 4 triangles ABB₁ | $\frac{1}{4}$ □ABCD | 1×1+4(△ABB₁) | $= 1 + 4(\frac{1}{4}$ □ABCD) $= 1 + 4 \times \frac{1}{4} \times 2 \times 2$ $= 5$ | 2×2+1×1 | Original Square Area + 1 unit² |
| 3 | 3×3 It has 6 triangles ABB₁ | $\frac{1}{6}$ □ABCD | 2×2+4(△ABB₁) | $= 4 + 4(\frac{1}{6}$ □ABCD) $= 2+4 \times \frac{1}{6} \times 3 \times 3$ $= 10$ | 3×3+1×1 | |
| 4 | 4×4 It has 8 ($\frac{1}{2 \times 4}$) triangles ABB₁ | $\frac{1}{8}$ □ABCD | 3×3+4(△ABB₁) | $= 9 + 4(\frac{1}{8}$ □ABCD $= 9 + 4 \times \frac{1}{8} \times 4 \times 4$ $= 17$ | 4×4+1×1 | |
| 5 | 5×5 It has 2×5 triangles ABB₁ | $\frac{1}{10}$ □ ABCD | 4×~~4~~4+4(△ABB₁) | $= 16 + 4(\frac{1}{10}$ □ ABCD $= 16+4 \times \frac{1}{10} \times 5 \times 5$ $= 26$ | 5×5+1×1 | |
| n | n×n It has 2n triangles ABB₁ | $\frac{1}{2n}$ □ABCD | (n-1)(n-1) ~~+4(△ABB₁)~~ $(n-1)(n-1) + 4(\frac{1}{2n}$ □ABCD) $= (n-1)^2 + 4 \times \frac{1}{2n} \times n \times n$ $= (n-1)^2 + 2n$ $= n^2 - 2n + 1 + 2n$ $= n^2+1$ | | $n^2+1 \times 1$ | CONJECTURE ABOUT THE AREA OF TILTED SQUARES is original AREA + 1 unit² |

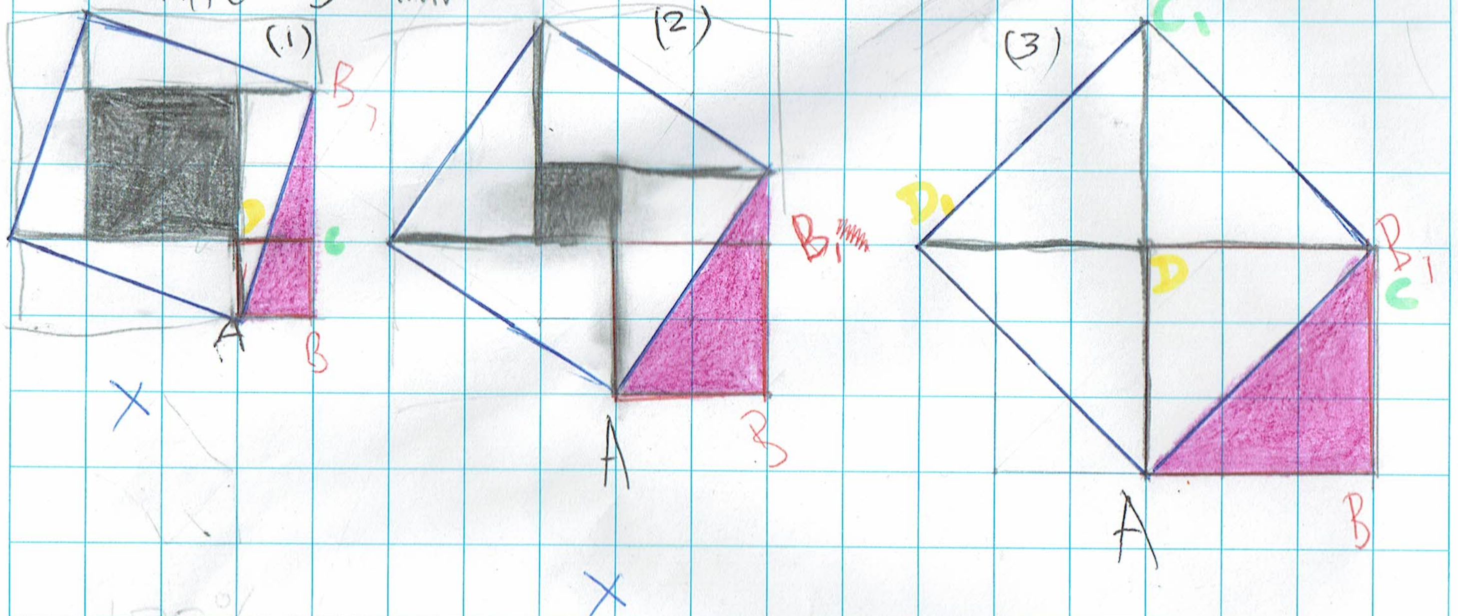
10mm Squares

# Tilted Squares

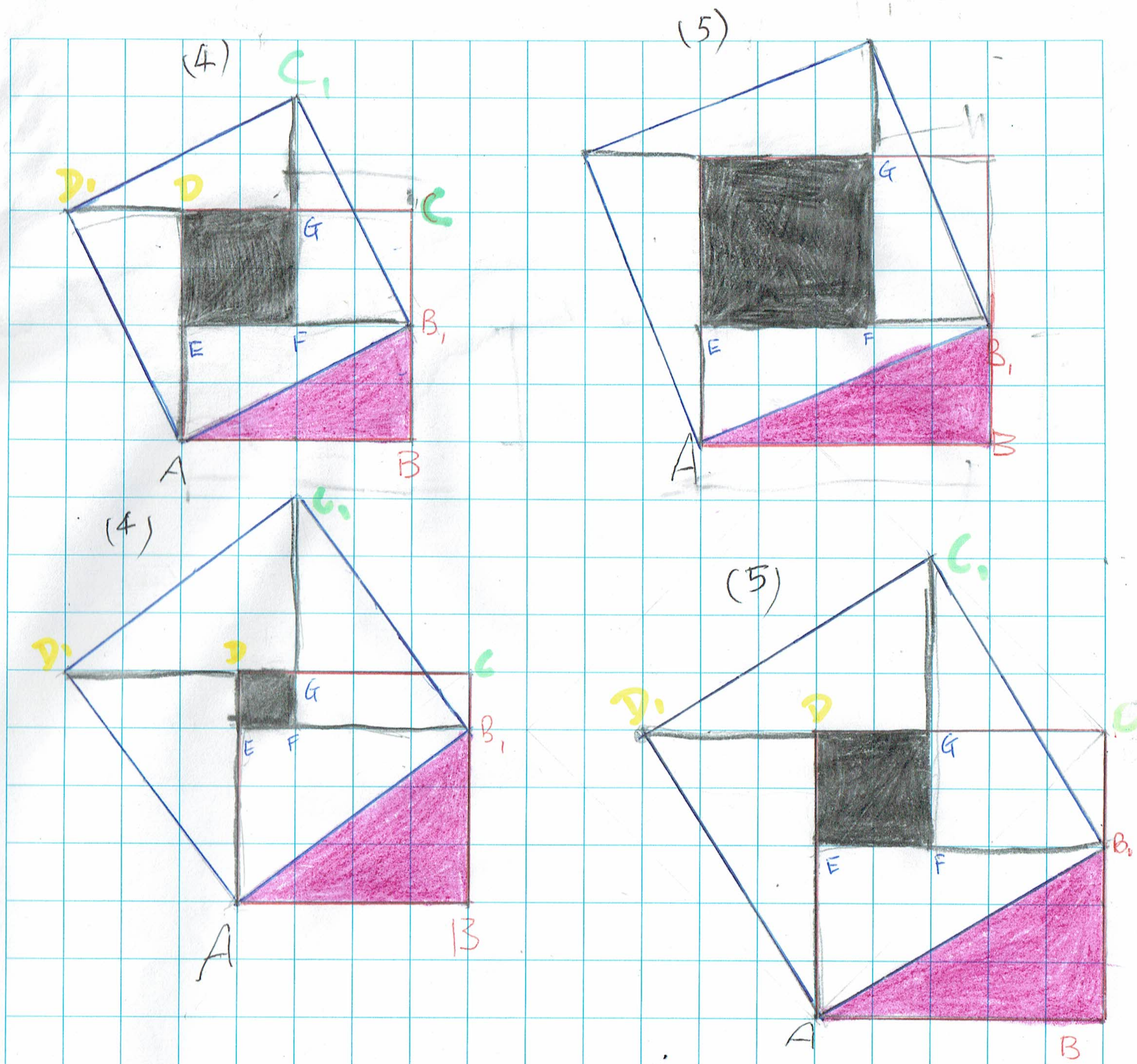
## Tilted 2 units




## Tilted 3 units




10mm Squares



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| | Tilted 1 unit | | | Tilted 2 units | | | Tilted 3 units | | | Tilted m units |
|---|---|---|---|---|---|---|---|---|---|---|
| Diagram | Area of original square ABCD | 1 $\Delta ABB_1$ = ? sq ABCD | | Area of original square ABCD | 1 $\Delta ABB_1$ = ? square ABCD | Area of tilted square $AB_1C_1D_1$ | Area of original square ABCD | 1 $\Delta ABB_1$ = ? square ABCD | Area of tilted square $AB_1C_1D_1$ | |
| 1 | 1x1 | $\frac{1}{2}$ | | | | | | | | |
| 2 | 2x2 | $\frac{1}{4}$ | | 2x2 | $\frac{1}{2}$ of sq ABCD | $(2-2)\times[2-2]$<br>$+4(\Delta ABB_1)$<br>$0\times0+4(\frac{1}{2}$ sq ABCD$)$<br>$0\times0+2$ sq ABCD | | | | |
| 3 | 3x3 | $\frac{1}{6}$ | Refer to P1/4 | 3x3 | $\frac{1}{3}$ sq ABCD | $(3-2)\times(3-2)+4(\Delta ABB_1)$<br>$1\times1+4(\frac{1}{3}$ sq ABCD$)$ | 3x3 | $\frac{1}{2}\frac{3}{3}$ sq ABCD | | Note: |
| 4 | 4x4 | $\frac{1}{8}$ | | 4x4 | $\frac{1}{4}$ sq ABCD | $(4-2)(4-2)+4(\Delta ABB_1)$<br>$2\times2+4(\frac{1}{4}$ sq ABCD$)$ | 4x4 | $\frac{1}{2}\frac{3}{4}$ sq ABCD | | |
| 5 | 5x5 | $\frac{1}{10}$ | | 5x5 | $\frac{1}{5}$ sq ABCD | $(5-2)(5-2)+4(\Delta ABB_1)$<br>$3\times3+4(\frac{1}{5}$ sq ABCD$)$ | 5x5 | $\frac{1}{2}\frac{3}{5}$ sq ABCD | | |
| n | nxn | $\frac{1}{2n}$ | $(n-1)(n-1)+4(\Delta ABB_1)$<br>$(n-1)^2+4\times\frac{1}{2n}\times n\times n$<br>$=n^2+1^2$ | nxn | $\frac{1}{n}$ sq ABCD | $(n-2)(n-2)+4(\Delta ABB_1)$<br>$(n-2)(n-2)+4(\frac{1}{n})(n\times n)$<br>$n^2+2^2$ | nxn | $\frac{1}{2}\frac{3}{n}$ sq ABCD | $(n-3)(n-3)$<br>$+4(\frac{1}{2}\times\frac{3}{n}\times n\times n)$<br>$n^2+3^2$ | $(n-m)(n-m)+4(\frac{1}{2}\times\frac{m}{n}\times n\times n)$<br>$=n^2+m^2$ |

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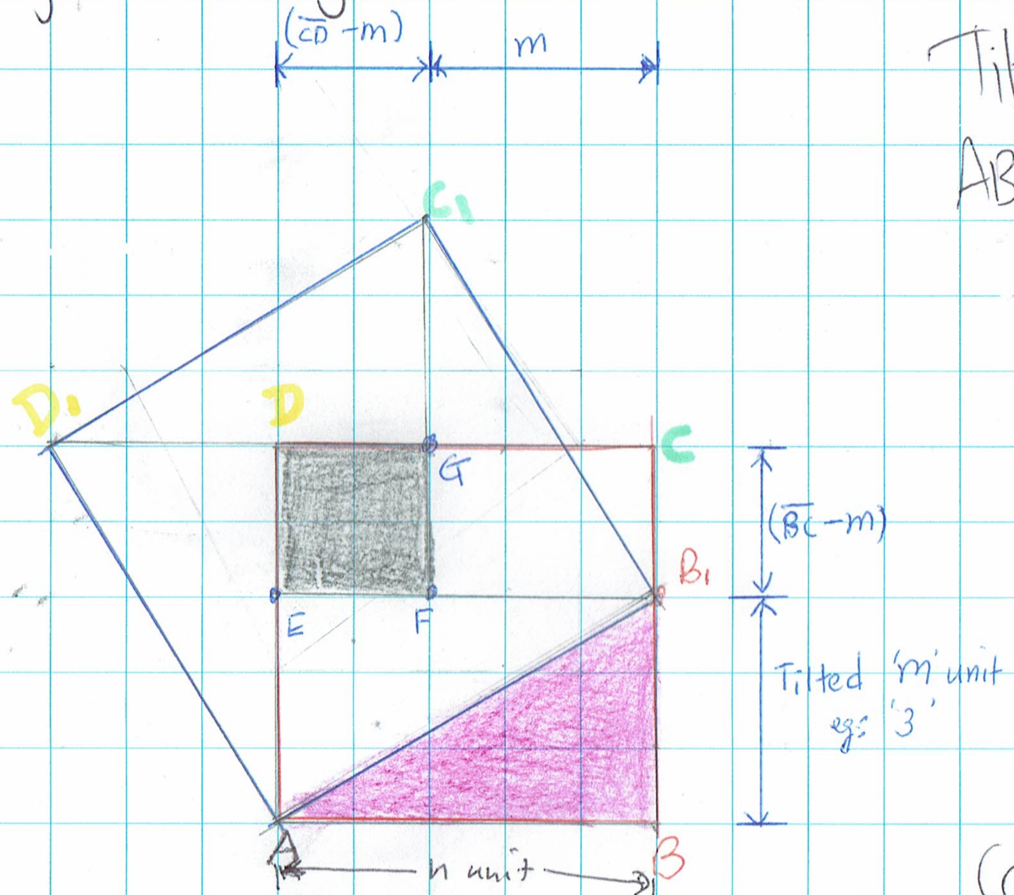
# Summary of investigations




$\Delta ABB_1$ is related to shape ABCD

Area of triangle $= \frac{1}{2}$ base × height

Base $= \overline{AB}$ Height $= \overline{BB_1}$ = Tilted 'm' unit

$\Delta ABB_1 = \frac{1}{2} \overline{AB} \times \overline{BB_1}$

$= \frac{1}{2} \overline{AB} \times m$

Ratio of $\frac{\Delta ABB_1}{ABCD} = \frac{\frac{1}{2}\overline{AB} \times m}{\overline{AB} \times \overline{BC}} = \frac{1}{2} \frac{m}{\overline{BC}}$

If ABCD is a $\boxed{sq}$, and $\overline{AB} = \overline{BC} = n$ units

$\Delta ABB_1 = \frac{1}{2}\left(\frac{nm}{nn}\right) sq\,ABCD$

$= \frac{1}{2} \frac{m}{n} sq ABCD$

## Tilted shape

$AB_1C_1D_1 = EFGD + \Delta B_1EA + \Delta C_1FB_1 + \Delta D_1GC_1 + \Delta ADD_1$ (All are Congruent Triangle & Triangle $ABB_1$)

$= EFGD + 4(\Delta ABB_1)$ — ①

$EFGD = \overline{EF} \times \overline{FG}$

$= (\overline{CD} - m) \times (\overline{BC} - m)$

$= (n-m)(n-m)$ ( ∵ square ABCD $\overline{CD} = \overline{BC} = \overline{AB} = n$ units ) — ②

Conjecture of tilted area related to original area (After tilted m unit):

$AB_1C_1D_1 = (n-m)(n-m) + 4(\Delta ABB_1)$ This will only work when $n \geq m$ (side $\overline{AB} = \overline{BC} = n$)

$= (n-m)^2 + 4(\frac{1}{2} \times \frac{m}{n} \times n \times n)$

$= (n^2 + m^2)$

I predict it will also work for rectangles. ♡

$AB_1C_1D_1 = (n_1 - m)(n_2 - m) + 4(\frac{1}{2}\frac{m}{n_2} \times n_1 \times n_2)$ (Sides: $\overline{AB} = n_1$, $\overline{BC} = n_2$)

$n_1 \geq m$

$n_2 \geq m$