

## NRICH Problem

### Pair Products

First example, four consecutive whole numbers: **2, 3, 4, and 5.**

First I multiplied the first and last numbers:  $2 * 5 = 10$ .

Then I multiplied the middle pair:  $3 * 4 = 12$ .

Then I tried another set: **8, 9, 10, and 11.**

I multiplied the first and last numbers:  $8 * 11 = 88$ .

Also multiplied the middle pair:  $9 * 10 = 90$ .

Another set I tried was: **30, 31, 32, 33.**

I multiplied the first and last numbers:  $30 * 33 = 990$ .

Then multiplied the middle pair:  $31 * 32 = 992$ .



From these examples, it seems that when I multiplied the first and last numbers in a set of four consecutive whole numbers, I got a product that is smaller than the product of the middle pair by 2.

This observation is true for other sets of four consecutive whole numbers as well, and it will always happen. To understand why this happens, I considered a formula for the four consecutive whole numbers:  $n, n+1, n+2,$  and  $n+3$ . Multiplying the first and last numbers gives us  $n(n+3) = n^2 + 3n$ . Multiplying the middle pair gives us  $(n+1)(n+2) = n^2 + 3n + 2$ .

We can see that the product of the middle pair includes an additional term (+2) compared to the product of the first and last numbers. This extra term causes the product of the middle pair to be greater than the product of the first and last numbers by 2. Therefore, it will always happen that when you multiply the first and last numbers in a set of four consecutive whole numbers, the result will be less than or equal to the product of the middle pair.