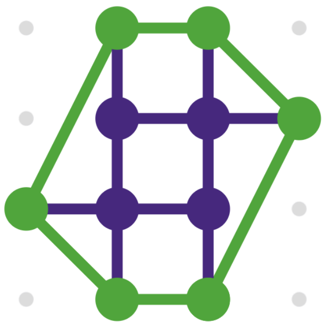
GANIT KREEDA MONDAY BATCH

I am Preethi Rao. I am one of the facilitators at **'Ganit Kreeda', Vichar Vatika, India**. I worked collaboratively with a group of students from 4th grade to 8th grade, online, on **"PICK's THEOREM".** The names of the students are:

Abheer, Anirved, Aslesha, Asma, Dia, Eshaan, Harshad, Harshika, Jaydeep, Nishika, and Siddhanth.

AREA OF THE POLYGONS

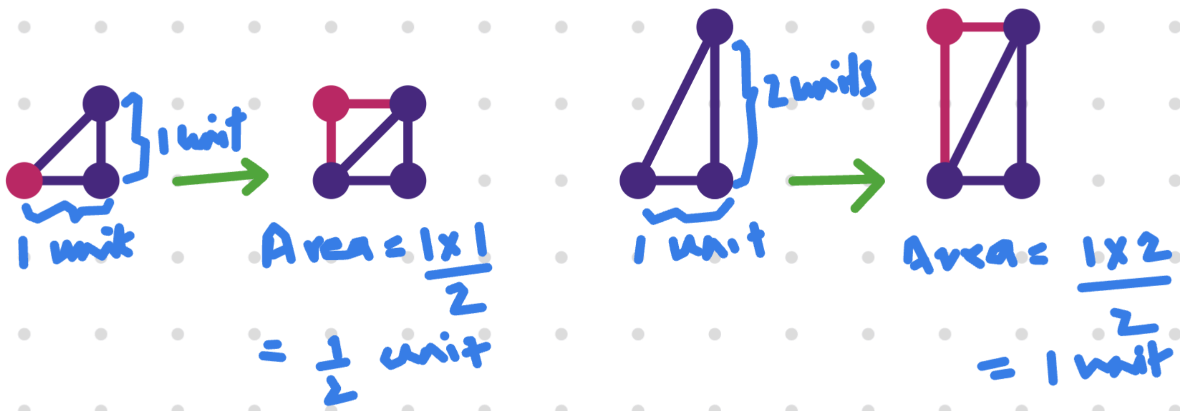
Children worked on figuring out how to calculate the area of the polygons. Children figured out that the entire polygon could be broken into squares, rectangles or triangles and the area of the individual shapes would add up to the area of the whole polygon.



Most of them knew the formula for finding the area of a square and rectangle. But did not know about triangles.

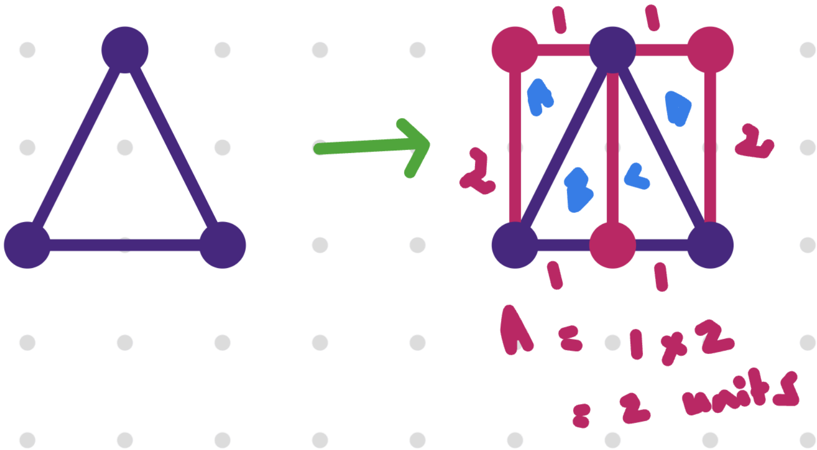
(i) Area of the Right Angled Triangle

Jaydeep, Asma and Nishika figured out that if a right angled triangle is doubled, then we get a rectangle. And so half the area of the rectangle = area of the right angled triangle.



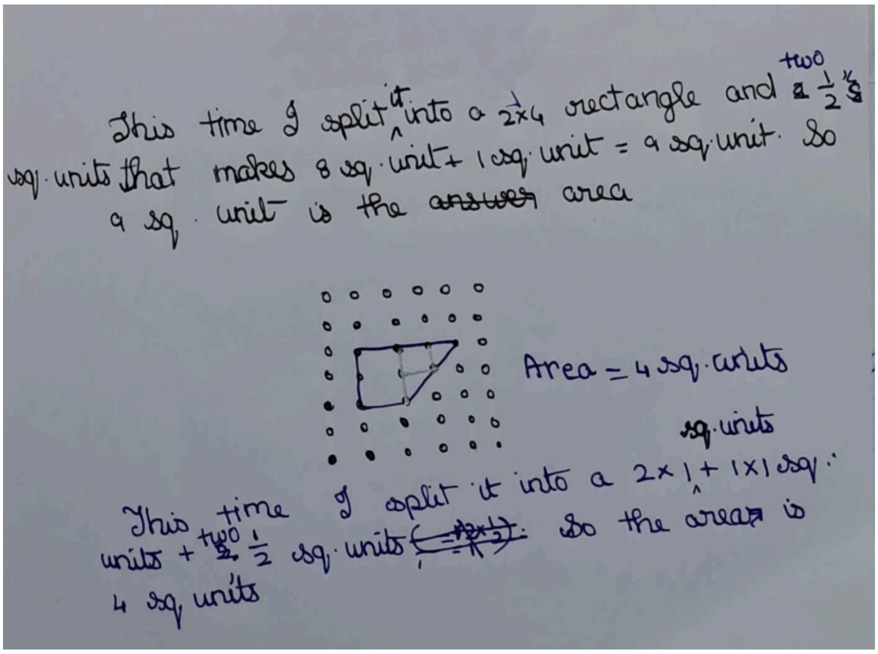
(ii) Area of the Isosceles and Scalene Triangle

The above idea was extended to the Isoceles triangle. Dia observed that if we drop one more line dividing the triangle, we have essentially 4 congruent triangles. And so the area of any one of the rectangles formed by triangle A and B or triangle C and D would give the area of the Isoceles triangle.

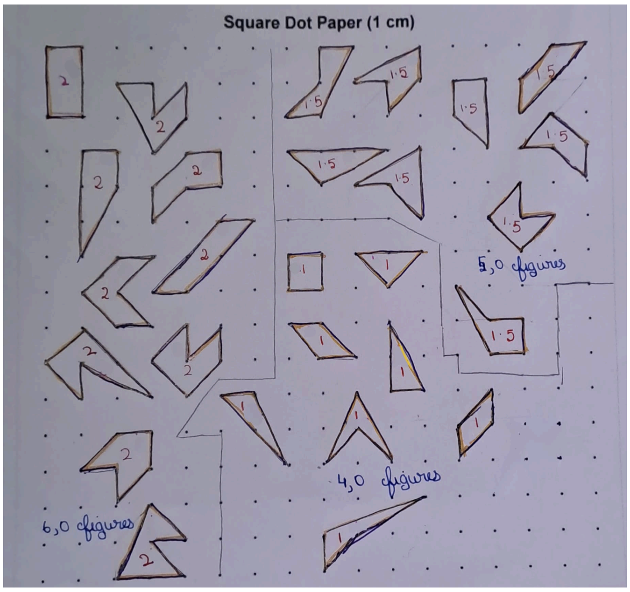


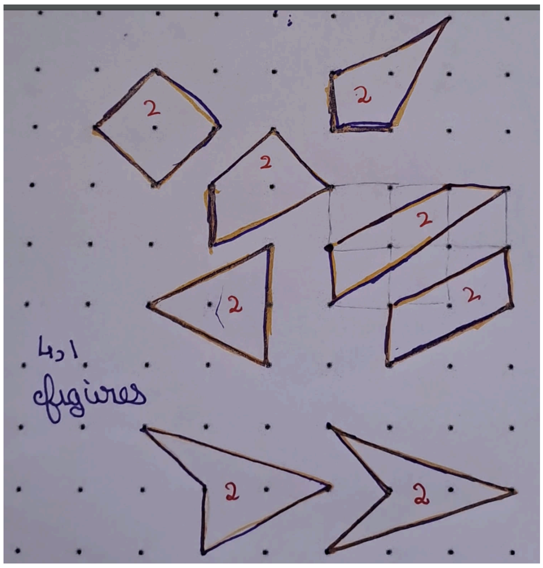
Children figured out the formula for area of the triangle = l x b / 2. They used this to find out the area of the scalene triangle after completing the rectangle

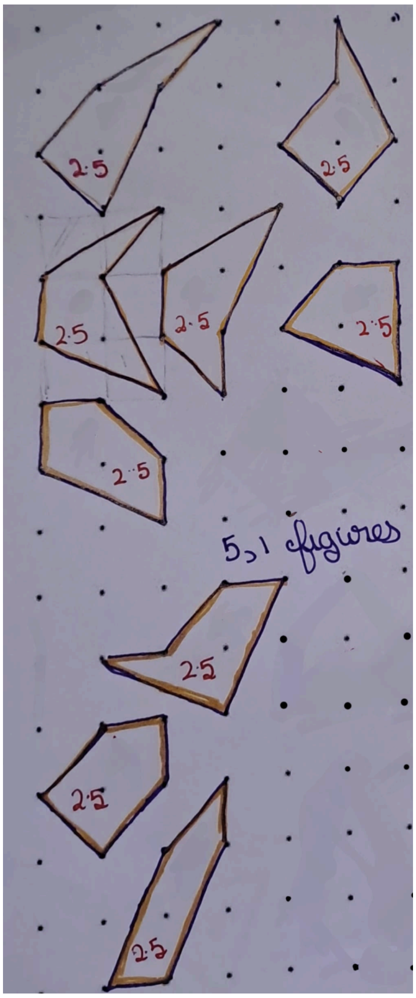
They calculated the areas of the given figures again:

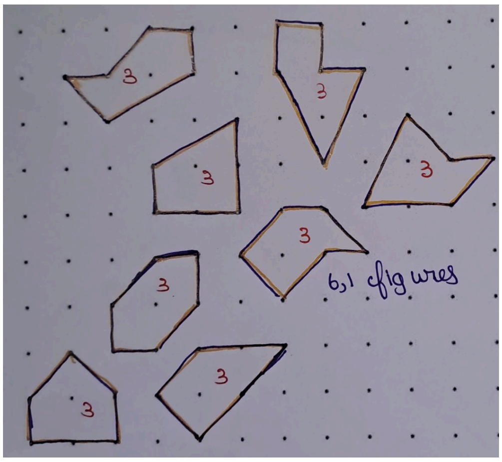


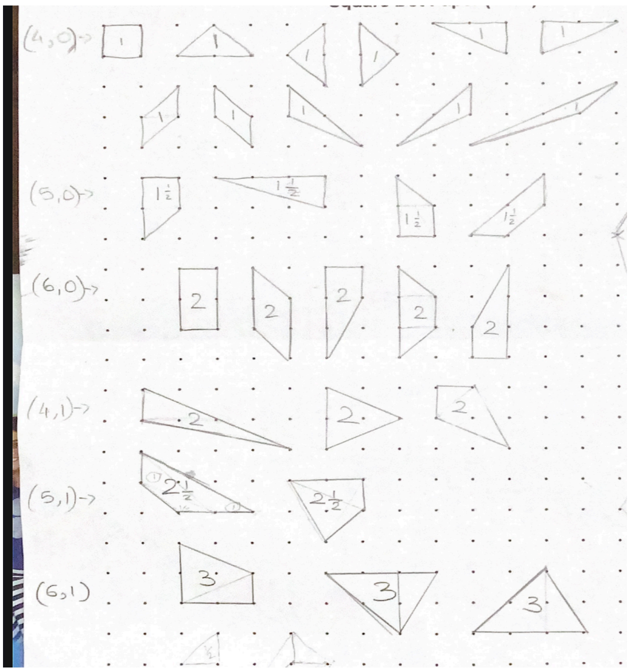
Children then drew figures with different perimeter points and interior points and noted down their areas.











Nishika noted down the perimeter points, interior points and the corresponding areas in a tabular fashion.

No. of dots on the border	No. of dots inside the figure	Area
4	0	1
5	0	$1\frac{1}{2}$
6	0	2
4	1	2
5	1	$2\frac{1}{2}$
6	1	3
4	2	3
3	0	$\frac{1}{2}$
3	1	$1\frac{1}{2}$

She noted down her observations:

Dots on the border	Area
4	1
5	$1\frac{1}{2}$
6	2

Relation - $\left(\text{No. of dots on the border} - 2\right) \div 2 = \text{Area}$

1) **Whenever the number of perimeter points increased by 1, the area increased by ½ unit. To figure out the contribution by the perimeter points Nishika just listed the perimeter points and the related area. She then observed the pattern and figured out the relationship.**
2) **Whenever the number of interior points increased by 1, the area increased by 1 unit. If there are 2 interior points, the area increases by 2 units…if there are three points, then the area increases by 3 units and so on….**

So from # 1 and # 2 , she arrived at PIck's theorem

$$\frac{P-2}{2} + I = \text{Area}$$

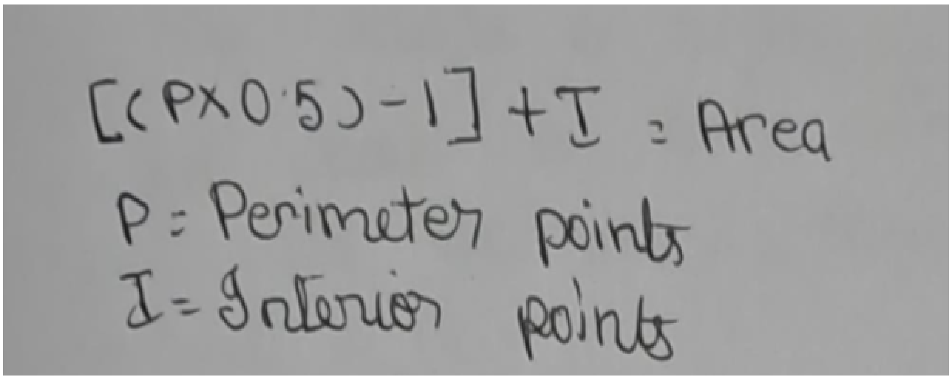
Generalized Method

$$\frac{\left[\text{No. of dots on the border} - 2\right]}{2} + \left(\text{No. of dots inside}\right) = \text{Area}$$

P = No of dots on the border
I = No of dots inside

Asma tabulated her results and based on her observations she came up with the same formula.

1) She noted that the increase of one perimeter point leads to an increase in the area by 0.5 units^2.
2) So she multiplied the Perimeter Points by 0.5. So if there are 3 perimeter points the area would be 3 * 0.5 = 1.5 units^2 .However, we have to discount the contribution by Perimeter points 1 and 2, because we cannot make a polygon with 1 perimeter point and 2 perimeter points. **So their contribution has to be subtracted i.e we have to subtract 1 unit^2 (0.5 unit^2 for each of the perimeter points).**
3) And so the contribution by perimeter points is **P*0.5 -1.**
4) She also observed that increase in one Interior point contributed to increase in 1 unit^2. Therefore if there are 2 interior points, area increases by 2 unit^2 and if there are 3 interior points the area increases by 3 unit^2 and so on ... .**So we have to just add the Interior point.**



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For the question "What do you notice about those figures whose areas are the same?" Anirved observed that if the Lattice points are increased by 2 and at the same time if the interior points are decreased by 1, then those figures share the same area.

The area of the figures of (3,1) and (3+2, 1-1) = (5,0) are same.

The figures of (4, 1) and (4+2, 1-1) = (6,0) are also equal.

So, the sets of $\boxed{(5, 1) \text{ and } (5+2, 1-1) = (7, 0)}$ set, $\boxed{(6,1) \text{ and } (8, 0)}$ and $\boxed{(7, 2) \text{ and } (9, 1)}$ all have an area of 2.5, 3 and 4.5 respectively.

Asma observed that

1) **the Lattice points are consecutive odd numbers or consecutive even numbers, if the polygons have the same area**
2) **And the Interior points differ by 1**

6 1 3.0 unit

1. The figures of the same area in this table are:

L = 3 I = 1, L = 5 I = 0 ; L = 4, I = 1, L = 6 I = 0

I observed that in these pairs the number of Latices point are either two consecutive odd or even number. I also observed that in ~~eath~~ each pair, the figures which have more latice points compared to the other figure in the pair have 0 interior points and the ~~figure to~~ other figure has 1 interior point.

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For the question “What ways are there of increasing the area by 1 unit?”

There are two ways to increase the area by 1 unit:

1) **The area increases by 0.5 unit^2 whenever the perimeter points are increased by 1. Therefore to get an increase of 1 unit^2 we have to increase the perimeter points by 2 while keeping the interior points the same.**
2) **The area increases by 1 unit^2 when we increase the interior point by 1. So we can do that while we keep the perimeter points the same.**

2. We can increase the area by 1 unit square by increasing the ~~interior~~ perimeter points by 2 because every time we increase the ~~interior~~ perimeter points by 1 it is $+0.5$ unit^2. So we need $0.5 \times 2 = 1$ unit^2. We can otherwise increase the interior points by 1 because every time I increased the interior points by 1 the whole area would increase by 1 unit^2. Example

P, I
5,0 = 1.5 unit^2
6,0 = 2.0 unit^2) +1 unit^2
7,0 = 2.5 units

- We can increase the interior points by 1 unit with the perimeter points being the same.
- We can increase the perimeter points by 2, the int. pts. ~~being~~ being the same.
- (+4, −1), (+6, −2), (+8, −3), etc. will also increase the area by 1.