Ci Hui Minh Angoc Ong Queensland Academies for Science Mathematics and technology (QASMT) Brisbane, Australia (age 12, grade 7)

16/6/2024

10mm Squares

P=perimeter points I=internal points A=area (P, I, A)

Pick's Theorem

10mm Squares

p1/3

P	I	A
3	0	$\frac{1}{2}$
4	0	1
5	0	$1\frac{1}{2}$
6	0	2
7	0	$2\frac{1}{2}$
8	0	3
9	0	$3\frac{1}{2}$
10	0	4
11	0	$4\frac{1}{2}$
12	0	5

observation:
across one column (I increases by 1) area (A) + 1 unit²
down one row A increases by $+\frac{1}{2}$ units² (as P increases by 1)

P	I	A
3	1	$1\frac{1}{2}$
4	1	2
5	1	$2\frac{1}{2}$
6	1	3
7	1	$3\frac{1}{2}$
8	1	4
9	1	$4\frac{1}{2}$
10	1	5
11	1	$5\frac{1}{2}$
12	1	6

P	I	A
3	2	$2\frac{1}{2}$
4	2	3
5	2	$3\frac{1}{2}$
6	2	4
7	2	$4\frac{1}{2}$
8	2	5
9	2	$5\frac{1}{2}$
10	2	6
11	2	$6\frac{1}{2}$
12	2	7

P	I	A
3	3	$3\frac{1}{2}$
4	3	4
5	3	$4\frac{1}{2}$
6	3	5
7	3	$5\frac{1}{2}$
8	3	6
9	3	$6\frac{1}{2}$
10	3	7
11	3	
12	3	

P	I	A
3	4	$4\frac{1}{2}$
4	4	5
5	4	$5\frac{1}{2}$
6	4	6
7	4	$6\frac{1}{2}$
8	4	7
9	4	$7\frac{1}{2}$
10	4	8
11	4	$8\frac{1}{2}$
12	4	9

I predict ↓

P	I	A
3	5	$5\frac{1}{2}$
4	5	6
5	5	$6\frac{1}{2}$
6	5	7
7	5	$7\frac{1}{2}$
8	5	8
9	5	$8\frac{1}{2}$
10	5	9
11	5	$9\frac{1}{2}$
12	5	10

1. Refer to diagrams in pages 2 & 3

2. $P \geq 3$ because you need 3 points to form an area (2 points form a line)

3. Minimum area is $\frac{1}{2}$ units² when $P=3$ and $I=0$

4. Find the expression of A enclosed by the number of perimeter (P) points and has a number of internal (I) points.

$A_{P,I}$ is the symbol used here representing area (units²)

refer to analysis table

Conclusion:

$$A_{P,I} = \frac{1}{2} + (P-3) \times \frac{1}{2} + I \quad (\text{units}^2)$$

I=4

(4, 4, 5) (5, 4, 5.5) (6, 4, 6) (7, 4, 6.5) (8, 4, 7) (9, 4, 7.5) (10, 4, 8) (11, 4, 8.5)

I=3

(3, 3, 3.5) (3, 4, 4.5) (4, 3, 4)

I=2

(4, 2, 3) (3, 2, 2.5) (5, 2, 3.5) (6, 2, 4) (7, 2, 4.5) (8, 2, 5) (9, 2, 5.5) (10, 2, 6)

(6, 3, 5) (5, 3, 4.5)

(10, 3, 7) (9, 3, 6.5) (8, 3, 6) (7, 3, 5.5) (6, 3, 5)

I=1

(5, 1, 2.5) (6, 1, 3) (7, 1, 3.5) (8, 1, 4) (9, 1, 4.5)

(3, 1, 1.5) (4, 1, 2)

(4, 0, 1)

I=0

(5, 0, 1.5) (6, 0, 2) (7, 0, 2.5) (8, 0, 3) (9, 0, 3.5)

(3, 0, 1.5)

LEGEND

(P I A)

perimeter points

internal points

area

: I=0 : I=3

: I=1 : I=4

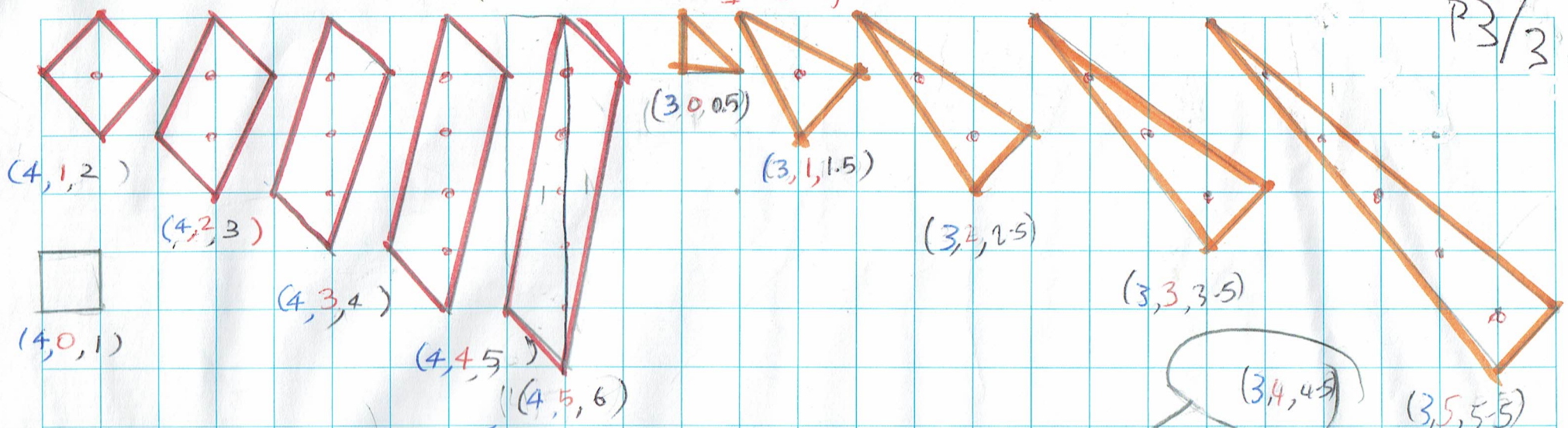
: I=2

10mm Squares

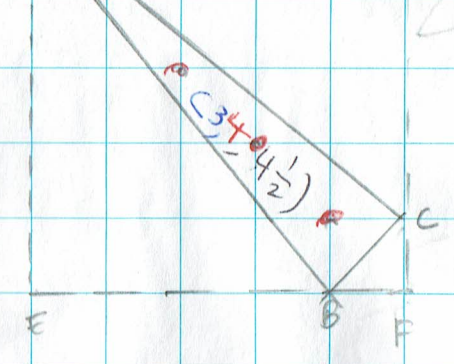
10mm Squares

P3/3

I = changes



Calculation Steps



area $\triangle ABC$ = □AEFD − △AEB − △BFC − △CAD

$= 5\times5 - \left(5\times4\times\frac{1}{2}\right)\times2 - \left(\frac{1}{2}(1\times1)\right)$

$= 4\frac{1}{2}$

Analysis table

$A_{P,I}$	$I=0$	$I=1$	$\rightarrow \dots$	I
$P=3$	$\frac{1}{2}$	$\frac{1}{2}+(1-0)\times 1$		$\frac{1}{2}+(I-0)\times 1$
$P=4$	$\frac{1}{2}+(4-3)\times\frac{1}{2}$	$\frac{1}{2}+(4-3)\times\frac{1}{2}$ $+(1-0)\times 1$		$\frac{1}{2}+(4-3)\times\frac{1}{2}$ $+(I-0)\times 1$
P	$\frac{1}{2}+(P-3)\times\frac{1}{2}$	$\frac{1}{2}+(P-3)\times\frac{1}{2}$ $+(1-0)\times 1$		$\frac{1}{2}+(P-3)\times\frac{1}{2}$ $+(I-0)\times 1$

$$A_{P,I} = \frac{1}{2}+(P-3)\times\frac{1}{2}+I \quad (\text{units}^2)$$