

Nrich Challenges

Pick's Theorem

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When $I = 0$:

Shape A: P I A
8 0 3

Shape B: P I A
7 0 $2\frac{1}{2}$

Shape C: P I A
6 0 2

$$P = 2A + 2 \quad \therefore 2 \times 3 + 2 = 8, \quad 2 \times 2\frac{1}{2} + 2 = 7, \quad 2 \times 2 + 2 = 6$$

When $I = 1$:

Shape D: P I A
6 1 3

Shape E: P I A
4 1 2

Shape F: P I A
7 1 $3\frac{1}{2}$

$$P = 2A \quad \therefore 2 \times 3 = 6, \quad 2 \times 2 = 4, \quad 2 \times 3\frac{1}{2} = 7$$

When $I = 2$:

Shape G: P I A
10 2 6

Shape H: P I A
11 2 $6\frac{1}{2}$

Shape I: P I A
4 2 3

$$P = 2A - 2 \quad \therefore 2 \times 6 - 2 = 10, \quad 2 \times 6\frac{1}{2} - 2 = 11, \quad 2 \times 3 - 2 = 4$$

When $I = 3$:

Shape J: P I A
12 3 8

Shape K: P I A
4 3 4

Shape L: P I A
10 3 7

$$P = 2A - 4 \quad \therefore 2 \times 8 - 4 = 12, \quad 2 \times 4 - 4 = 4, \quad 2 \times 7 - 4 = 10$$

In conclusion, the relation-ship between PIA is $P = 2A - 2(I - 1)$

\therefore When $I = 0$, ~~$P = 2A$~~ $P = 2A - 2(0 - 1)$

$$P = 2A + 2$$

When $I = 1$, $P = 2A - 2(1 - 1)$

$$P = 2A$$

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