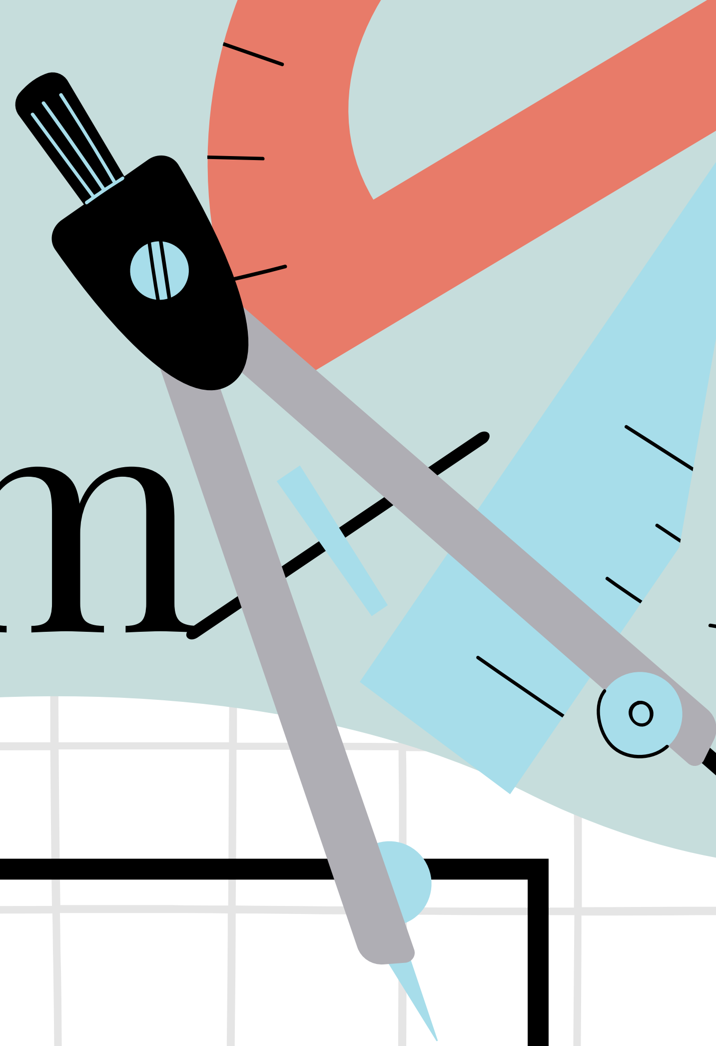


Pick's Theorem

Intro & Pre-Data

Introduction

This problem will require tabulation of data. This is because if we tabulate the data, we will be able to find the solution. The goal is a 10*10 table for P (perimeter dots) and I (internal dots), with table values of A (area).

We can then proceed to analyze the data, before creating a relationship between the perimeter, internal dots, and the area.

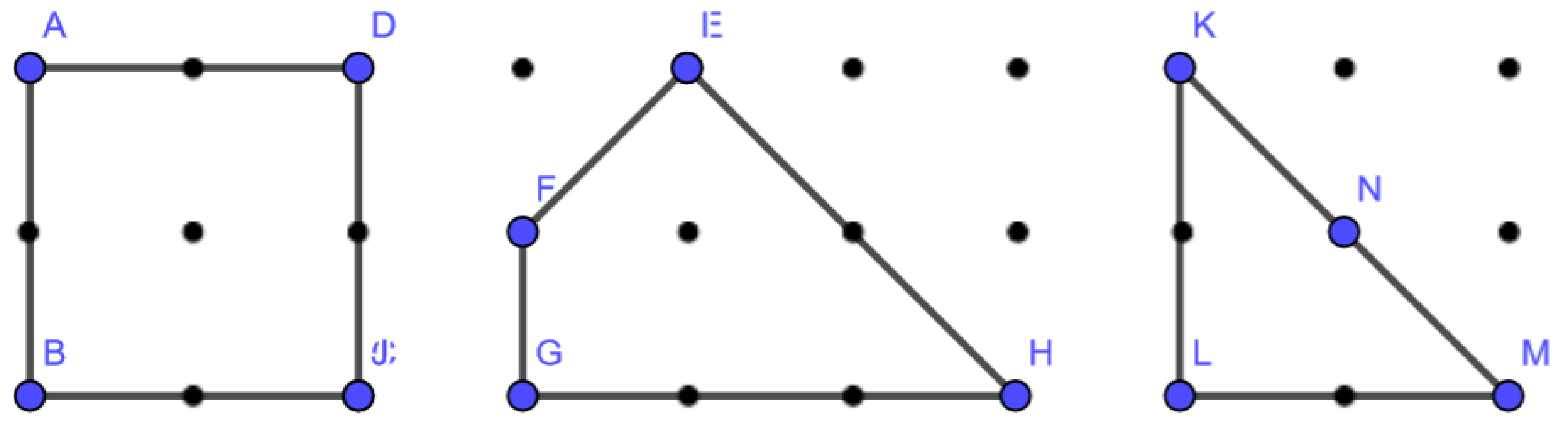
Pre-Data collection

Before collecting the data, we must first ensure that each shape with the same amount of perimeter and internal points have the same area; otherwise, data collection would become virtually impossible.

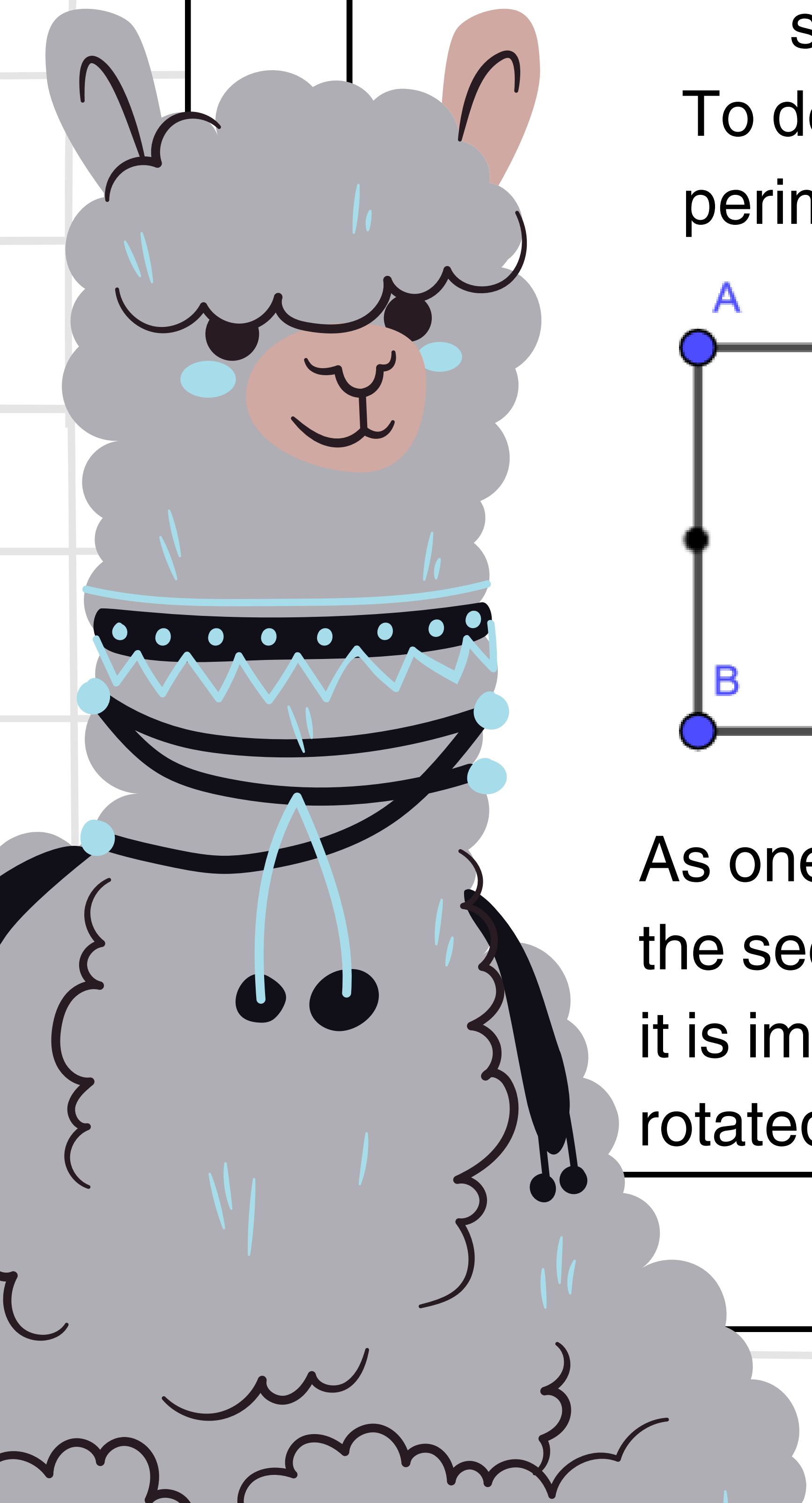
There are 2 possibilities:

- Each amount of internal and external dots results in a different area.
- Each amount of internal and external dots results in a different shape or ratio between the perimeter dots.

To do this, we will atttempt to find a shape with the same amount of perimeter and internal dots:



As one can see, this is impossible. The first image has a (*p*,*i*) of 8,1, the second 7,1, and the last 7,0. As each vertex has to be on a point, it is impossible for 2 shapes to share a value of p & i, unless it is a rotated variant of the previous shape.



Data Collection

Context

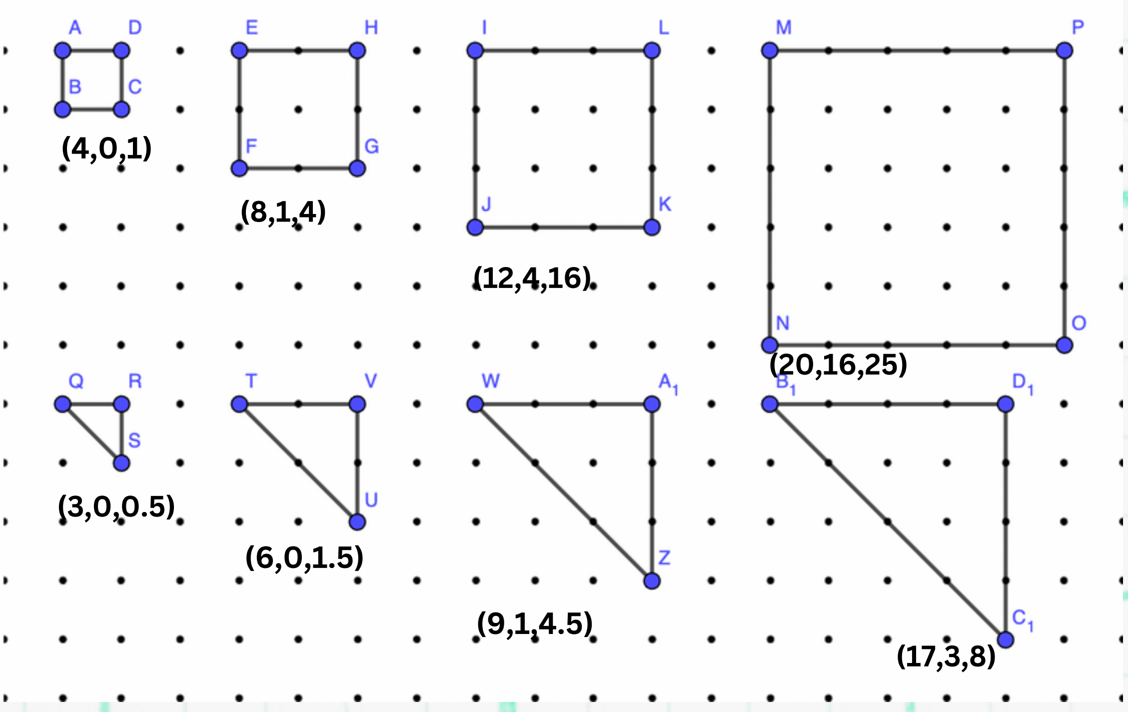
How do we tabulate the data

We can tabulate the data by finding the *p* (perimeter points) and *i* (internal points) amounts.

We can then find the area of the shape.

The *p* value starts at 3, while the *i* value starts at 0.

Below are some examples. The first value is *p,* the second *i*, and the last the area (*a*).



Table

p

i

	3	4	5	6	7	8	9
0	0.5	1	1.5	2	2.5	3	3.5
1	1.5	2	2.5	3	3.5	4	4.5
2	2.5	3	3.5	4	4.5	5	5.5
3	3.5	4	4.5	5	5.5	6	6.5
4	4.5	5	5.5	6	6.5	7	7.5
5	5.5	6	6.5	7	7.5	8	8.5

Analysis

Results

It appears that each time *p* increases, the area increases by 0.5, while each time *i* increases, the area increases by 1. P is 2 at area 0, so a formula for the area with p and i as inputs would be:

$$a(p, i) \ = \ ((p \ * \ 0.5) \ - \ 1) \ + \ i$$

The reason for subtracting 1 from p times 0.5 is so p can be 3 at area 0.5:

1.5 - 1 = 0.5