Tiny Nines

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Sunday, 22 March 2020

Generalising from patterns

$$a = \frac{1}{\underbrace{99\ldots99}_{k}}$$

$$a = \frac{1}{9} \cdot \frac{1}{\underbrace{11\ldots11}_{k}}$$

$$a = \frac{1}{9} \cdot \frac{1}{\underbrace{11\ldots11}_{k}}$$

$$a = \frac{1}{9} \cdot 0.\underbrace{\dot{0}00\ldots\dot{9}}_{k}$$ [Upon Observation]

$$a = \frac{1}{\cancel{9}} \cdot 0.\underbrace{\dot{0}00\ldots\dot{\cancel{9}}}_{k}$$

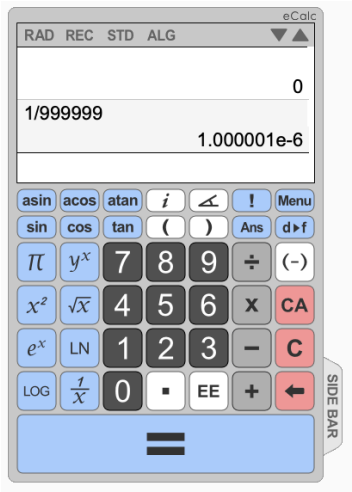
$$a = 0.\underbrace{\dot{0}00\ldots\dot{1}}_{k}$$

$$\frac{1}{\underbrace{99\ldots99}_{k}} = 0.\underbrace{\dot{0}00\ldots\dot{1}}_{k}$$

Examples:

$$a = \frac{1}{\underbrace{999999}_{6}}$$

Should equal $0.\underbrace{\dot{0}0000\dot{1}}_{6}$



$\frac{1}{999999}$ indeed equals $\dot{0}0000\dot{1}$

Extended 1:

$$\frac{1}{\underbrace{9}_{1}} = 0.\underbrace{\dot{1}}_{1}$$

$$\frac{1}{\underbrace{99}_{2}} = 0.\underbrace{\dot{0}\dot{1}}_{2}$$

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$$\frac{1}{\underbrace{99\ldots99}_{k}} = 0.\underbrace{\dot{0}00\ldots\dot{1}}_{k}$$

$$\underbrace{\frac{1}{\underbrace{9}_{1}}}_{} \times 3 = 0.\underbrace{\dot{1}}_{1} \times 3$$

$$\underbrace{\frac{1}{\underbrace{3}_{1}}}_{} = 0.\underbrace{\dot{3}}_{1}$$

$$\underbrace{\frac{1}{\underbrace{99}_{2}}}_{} \times 3 = 0.\underbrace{\dot{0}\dot{1}}_{2} \times 3$$

$$\underbrace{\frac{1}{\underbrace{33}_{2}}}_{} = 0.\underbrace{\dot{0}\dot{3}}_{2}$$

$$\underbrace{\frac{1}{\underbrace{33\ldots33}_{k}}}_{} = 0.\underbrace{\dot{0}00\ldots\dot{3}}_{k}$$

Extended 2:

$$\underbrace{\frac{1}{\underbrace{9}_{1}}}_{} \times 9 = 0.\underbrace{\dot{1}}_{1} \times 9$$

$$1 = 1$$

$$\underbrace{\frac{1}{\underbrace{99}_{2}}}_{} \times 9 = 0.\underbrace{\dot{0}\dot{1}}_{2} \times 9$$

$$\frac{1}{\underbrace{11}_{2}} = 0.\underbrace{\dot{0}\dot{9}}_{2}$$

$$\frac{1}{\underbrace{11\ldots11}_{k}} = 0.\underbrace{\dot{0}00\ldots\dot{9}}_{k}$$

Extended 3 Generalised:

$$= \frac{n}{\underbrace{99\ldots99}_{k}}$$

$$\frac{n}{\underbrace{99\ldots99}_{k}} = n \times 0.\underbrace{\dot{0}00\ldots\dot{1}}_{k}$$

SHORTCUT: Make 'n' the same digits long as 'k' by adding zeroes in front of n

$$\frac{n}{\underbrace{99\ldots99}_{k}} = 0.\underbrace{\dot{n}}_{k}$$

Extended 3 (Example for numerator 'n'):

Digits of n < k:

Example

$$= \frac{57}{999}$$

$$= 57 \cdot \frac{1}{999}$$

$$= 57 \times 0.\dot{0}0\dot{1}$$

$$= 57 \times 0.001001001001001...$$

$$= 0.057057057057$$

$$= 0.\dot{0}5\dot{7}$$

Digits of n = k:

Example

$$= \frac{23}{99}$$

$$= 23 \cdot \frac{1}{99}$$

$$= 23 \times 0.\dot{0}\dot{1}$$

$$= 23 \times 0.0101010101...$$

$$= 23 \times$$
$$0.01000000000000....$$
$$0.00010000000000....$$
$$0.00000100000000....$$
$$0.00000001000000....$$

$$= 0.232323232323...$$

$$= 0.\dot{2}\dot{3}$$

Digits of n > k:

Example

$$= \frac{320}{99}$$

$$= \frac{320}{99}$$

$$= 3 + \frac{23}{99}$$

$$= 3 + 0.\dot{2}\dot{3}$$

$$= 3.\dot{2}\dot{3}$$