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# Remainders

**I'm thinking of a number.**

**My number is both a multiple of 5 and a multiple of 6.**

**What could my number be?**

We require a number that is both a multiple of 5 and 6. Therefore one possibility for this number could be the lowest common multiple of 5 and 6, which is 30.

**What else could it be?**

It could also be any multiple of 30 because both 5 and 6 are factors of 30.

**What is the smallest number it could be?**

The smallest number is known as the lowest common multiple which is 30.

**I'm thinking of a number.**

**My number is a multiple of 4, 5 and 6.**

**What could my number be?**

To work out a potential number which is a multiple of 4,5 and 6, we need to consider the multiples of these individual numbers. Making a table of the first 15 multiples of 4,5 and 6 gives:

| Multiplying number | Multiples of 4 | Multiples of 5 | Multiples of 6 |
|---|---|---|---|
| 1 | 4 | 5 | 6 |
| 2 | 8 | 10 | 12 |
| 3 | 12 | 15 | 18 |
| 4 | 16 | 20 | 24 |
| 5 | 20 | 25 | 30 |
| 6 | 24 | 30 | 36 |
| 7 | 28 | 35 | 42 |
| 8 | 32 | 40 | 48 |

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|---|---|---|---|
| 9 | 36 | 45 | 54 |
| 10 | 40 | 50 | **60** |
| 11 | 44 | 55 | 66 |
| 12 | 48 | **60** | 72 |
| 13 | 52 | 65 | 78 |
| 14 | 56 | 70 | 84 |
| 15 | **60** | 75 | 90 |

If we carefully look at this table, the lowest common multiple of 4,5 and 6 is 60, which is in bold.

**What else could it be?**

Like the first example, this number could also be any multiple of 60.

**What is the smallest number it could be?**

The smallest number, known as the lowest common multiple, as I've shown in the table is 60.

**Here are some more questions you might like to consider:**

**I'm thinking of a number that is 1 more than a multiple of 7.**

**My friend is thinking of a number that is 1 more than a multiple of 4.**

**Could we be thinking of the same number?**

Using algebra, we can form two expressions for these numbers:

For one more than a multiple of 7 we get $7x + 1$, where x is a whole number.

For one more than a multiple of 4 we get $4x + 1$, where x is a whole number.

If we want both these people to be thinking of the same number, we can substitution of the 4 and 7 into the two expressions separately because they have the same constant of 1 added to them: substituting $x = 4$ into the first expression which would then equal $7(4) + 1 = 29$ and

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$x = 7$ into the second expression which gives $4(7) + 1 = 29$. therefore 29 is the smallest same number the two people could be thinking of.

If we look at all the 180 numbers in the number sieve, the numbers which could both could be thinking of which are the same are 57,85,113,141and 169.

**I'm thinking of a number that is 3 more than a multiple of 5.**

**My friend is thinking of a number that is 8 more than a multiple of 10.**

**Could we be thinking of the same number?**

8 more than a multiple of 10 is the same as 3 more than a multiple of 5. However, for every two multiples of 10 we have three multiples of 5. Because of this we know that any number ending in 8 could be a number both people are thinking of :

Using the 180 numbers of the number sieve this would be :
8,18,28,38,48,58,68,78,88,98,108,118,128,138,148,158,168,178.

**I'm thinking of a number that is 3 more than a multiple of 6.**

**My friend is thinking of a number that is 2 more than a multiple of 4.**

**Could we be thinking of the same number?**

No, the two people can never be thinking of the same number. This is because if we look at the sequence of the last digits of the sequence of numbers which are three more than a multiple of 6 we get 9,5,1,7,3 and this repeats. The last digits of the sequence of numbers which are two more than a multiple of 4 are 6,0,4,8,2. The last digits for 3 more than a multiple of 6 are always odd whereas the last digit for 2 more than a multiple of 4 are always even. Because of this, they can never think of the same number.

**Here's a challenging extension:**

**We know that**

**When 59 is divided by 5, the remainder is 4**

**When 59 is divided by 4, the remainder is 3**

**When 59 is divided by 3, the remainder is 2**

**When 59 is divided by 2, the remainder is 1**

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**Can you find a number with the property that when it is divided by each of the numbers 2 to 10, the remainder is always one less than the number it is has been divided by?**

**Can you find the smallest number that satisfies this condition?**

Firstly, for this number to produce a remainder of 9 when divided by 10, the end digit must be 9. Having figured this out, I decided to look at the multiples of 9, in particular ones in the sequence 81,171261,351 … . Then i would test whether one more than these numbers was a multiple of 8, if so then i'd test whether one more than the resulting number was a multiple of 7. The first number that caused this was 2511 : 2511/9 = 279 , 2512/8 = 314 , 2513/7 = 359 … . Therefore the smallest number that satisfies the above conditions is 2510+9 = **2519**

2519/10 = 251 **r 9**

2519/9 = 279 **r 8**

2519/8 = 314 **r 7**

2519/7 = 359 **r 6**

2519/6 = 419 **r 5**

2519/5 = 503 **r 4**

2519/4 = 629 **r 3**

2519/3 = 839 **r 2**

2519/2 = 1259 **r 1**